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Deciphering Dollar Exchange Rates and Interest Parity

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DECIPHERING DOLLAR EXCHANGE RATES AND INTEREST PARITY*

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Abstract

This paper explores exchange rate dynamics and the uncovered interest parity (UIP) violation in the context of multiple shocks. Our key contribution lies in revealing that exchange rate dynamics emanate from the collective influence of different shocks, in contrast to prevailing literature emphasizing the dominance of a single shock. While verifying the unconditional UIP reversals, we are the first to show that there is no significant evidence of conditional UIP reversal with an innovative test method developed in this paper. Additionally, through rigorous mathematical proof, we establish that conditional UIP reversal is not a prerequisite for unconditional UIP reversal in models featuring a moving averaging representation. This insight relaxes stringent prerequisites in earlier theoretical studies, offering broad applicability for understanding reversal patterns in UIP and other asset returns.

Keywords: Bayesian SVAR; Exchange rate; New Fama puzzle; Uncovered interest parity

JEL Codes: E32, E44, E52, F31, F41

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1 Introduction

The study of exchange rate movements is a central focus in the realm of international economics. Pioneered by Eichenbaum and Evans (1995), a large body of literature has been established to explore what drives exchange rate movements based on structural vector autoregression (SVAR) models. However, the literature has not yet reached a consensus on this issue and has proposed at least four competing explanations. Earlier studies highlight the role of monetary shocks (e.g., Kim and Roubini 2000, Scholl and Uhlig 2008, Kim et al. 2017, Rüth 2020). By contrast, Nam and Wang (2015), Klein and Linnemann (2021), and Chahrour et al. (2023) argue that anticipated technology shocks are predominant in explaining the exchange rate variations. Echoing the theoretical work of Chen (2021), Eichenbaum et al. (2021) and Itskhoki and Mukhin (2021a), recent papers (Georgiadis et al. 2021 and Jiang et al. 2021) point towards a special role of financial shocks that affect exchange rates through their impacts on safe asset demand. More recently, Schmitt-Grohé and Uribe (2022) ascribe much of the exchange rate volatility to shocks that trigger remarkably persistent interest rate responses.

Although the literature is far from speaking in one voice about what factors drive exchange rate dynamics, previous studies tend to highlight one of the competing explanations by focusing on a single dominant shock. A potential drawback of focusing on one shock at a time is the inherent risk that the effects of the identified shock may become confounded with other shocks so that the shock highlighted as the primary driver of exchange rates in one study may exhibit correlation with, or even correspond to, the shock emphasized in another paper. Additionally, the theoretical literature, exemplified by Itskhoki and Mukhin (2021a), endeavors to construct a unified framework aligning unconditional empirical moments with theoretical moments conditional on a single shock. Recent contributions, such as Itskhoki and Mukhin (2024), eliminate certain shocks as exchange rate drivers due to discrepancies between moments conditional on these shocks (one at a time) and unconditional moments. The analytical procedure undertaken in Itskhoki and Mukhin (2024) is valid under the assumption that the exchange rate as well as home and foreign interest rates are governed by a single shock, wherein the concurrence of conditional and unconditional moments is presumed. However, such elimination can be inappropriate when exchange rates are influenced by multiple shocks so that conditional moments may diverge from unconditional moments. Critically, the literature currently lacks empirical establishment of conditional moments and exploration of the relationship between conditional and unconditional moments.

To crack these challenges, this paper makes the first endeavor to jointly identify anticipated technology shocks, monetary shocks, safe asset demand shocks, and persistent interest rate shocks and let them compete in one unified open-economy SVAR framework. Our objectives encompass addressing several key research questions: Firstly, does the movement of dollar exchange rates predominantly stem from a singular type of shock, or is it influenced by the collective impact of multiple shocks? Secondly, how does our identification strategy offer insights into the persisting puzzles surrounding exchange rates and interest rate parity, as assessed through conditional moments? Lastly, can the examination of the relationship between conditional and unconditional moments shed on existing theoretical studies?

We estimate a seven-variable SVAR model with a Bayesian approach using quarterly data from 1975Q1 to 2018Q4. Our empirical analysis yields the following novel findings. First, the dynamics of the dollar exchange rates result from the combined influence of all identified shocks. This challenges the perspective that exchange rates are primarily influenced by a single type of shocks, whether it be financial shocks as proposed by Itskhoki and Mukhin (2021a), technology shocks according to Chahrour et al. (2023), or main foreign exchange rate shocks as identified by Miyamoto et al. (2023). Anticipated technology shocks and persistent interest rate shocks contribute more to exchange rate dynamics than monetary shocks and safe asset demand shocks. Nevertheless, safe asset demand shocks were responsible for much of the dollar appreciation during the global financial crisis in 2008.

Second, we illustrate that exchange rates respond to structural shocks in distinct patterns with disparate magnitudes. The impulse responses of exchange rates resemble a horizontal J-curve following anticipated technology shocks, an S-curve following monetary shocks, a hump-shaped curve following safe asset demand shocks, and an L-curve following persistent interest rate shocks. Exchange rates appreciate and overshoot immediately in response to safe asset demand shocks. By contrast, the overshooting of exchange rates is delayed for more than two years following the other three shocks. The observation that more persistent shocks play a pivotal role in elucidating exchange rate movements aligns with the perspective of Engel and West (2005), who contend that the near-random walk behavior of exchange rates can be rationalized by the persistent movements in macro fundamentals.

Third, we provide complementary empirical evidence to evaluate whether uncovered interest parity (UIP), both conditional and unconditional, holds between the US and the G6 countries. Using a dynamic version of the Fama (1984) regression following Engel (2016), we confirm that unconditional UIP fails and the co-movement between foreign exchange excess returns and cross-country interest differentials changes direction as time horizon extends. Moreover, we propose a novel approach to test the conditional UIP by implementing the dynamic Fama regressions with counterfactual data and model-implied conditional forecasts. Our new test indicates that the UIP fails conditional on monetary shocks and persistent interest rate shocks. There is no sufficient evidence to reject the conditional UIP reversal at long horizons.

In light of the empirical evidence, we further explore the feasibility of constructing a theoretical model demonstrating unconditional UIP reversal without exhibiting such reversal under any exogenous shocks. We mathematically prove that the unconditional dynamic Fama coefficients are a weighted average of conditional Fama coefficients, with the weight determined by the shocks' contribution to the cross-country interest rate differential.[1] Consequently, unconditional UIP reversal can occur when Fama coefficients

[1]Prior research typically asserts the significance of a shock in elucidating exchange rate dynamics when the moments conditional on a shock align with the unconditional dynamics. For instance, Itskhoki and Mukhin (2021a) argue that "if financial shocks play an important role in the dynamics of the exchange rate, the model reproduces a negative unconditional Fama coefficient." Our analysis challenges this notion,

conditional on two shocks have different signs without a discernible reversal pattern. This insight is applied to recent studies by Valchev (2020) and Candian and De Leo (2023). Valchev (2020) establishes conditions for unconditional UIP reversal, reliant on active monetary and persistent fiscal policy in an open-economy New Keynesian model, which our findings demonstrate can be relaxed if conditional UIP reversal is not a prerequisite. Similarly, Candian and De Leo (2023)'s proposed mechanism for both conditional and unconditional UIP reversal, driven by over-extrapolative belief and shock mis-perception, is adapted in our analysis. We show that replacing their shocks with the four shocks from our empirical study requires only one of the over-extrapolative belief and shock mis-perception to generate unconditional UIP reversal. In essence, our paper offers a new explanation for UIP reversal through the collective impact of multiple shocks, eliminating the reliance on a specific shock for its occurrence.

The overview of the literature helps place our contributions in context. First, we are the first that jointly identifies anticipated technology shocks, monetary shocks, safe asset demand shocks, and persistent interest rate shocks in an open-economy SVAR model. The innovative approach enables us to disentangle the effects of each of these shocks and their transmission channels under a unified framework with different sources of prior information. Wolf (2020) cautions the literature about the potential emergence of the "masquerading problem" in the SVAR model when employing a set identification approach to identify a single shock.

However, little efforts have been made to disentangle the effects of anticipated technology shocks and persistent interest rate shocks when identifying monetary shocks in open-economy frameworks. Our paper bridges this gap in the literature.

Second, while the literature considers the failure and reversal of UIP as unconditional moments in data, the empirical literature noticeably lacks comprehensive discussions on the dynamic pattern of UIP violation and reversal conditional on shocks. We introduce a novel method to examine conditional UIP by applying the dynamic Fama

asserting that conditional Fama coefficients can closely resemble their unconditional counterparts when conditioned shocks are influential on interest rate differentials rather than exchange rates.

regressions proposed by Engel (2016) to counterfactual data and conditional forecasts.[2] In contrast to the existing UIP test relying on excess return impulse responses following Eichenbaum and Evans (1995), aligns more coherently with the unconditional UIP test and is conceptually consistent with the conditional dynamic coefficients evaluated in some prior theoretical studies (Valchev 2020 and Candian and De Leo 2023). Moreover, our methodology is readily applicable to producing dynamic Fama coefficients (and other moments) conditional on multiple shocks simultaneously. This aspect, largely overlooked in prior empirical literature, proves valuable in assessing whether the observed unconditional moments can be generated by the convolution of multiple shocks.

Third, in contrast to prevailing theoretical literature (Valchev 2020, Na and Xie 2022, Candian and De Leo 2023), which often elucidates unconditional UIP reversal by designing mechanisms to achieve it conditional on specific shocks, our contribution lies in establishing, through mathematical proof, that unconditional Fama coefficients are weighted averages of conditional Fama coefficients. As a result, conditional UIP reversal is not a prerequisite for unconditional UIP reversal. Utilizing models (with variations) derived from Valchev (2020) and Candian and De Leo (2023), we demonstrate that the stringent conditions outlined in earlier models can be relaxed when conditional UIP is not a compulsory precursor to unconditional UIP reversal.

While our primary focus is on dynamic Fama coefficients, our analytical framework can be extended to establish various other conditional moments related to the exchange rate disconnect puzzles documented in Itskhoki and Mukhin (2021a).[3] Numerous prior studies have concentrated on generating theoretical conditional moments to match with empirical unconditional moments (Itskhoki and Mukhin 2021a, Itskhoki and Mukhin 2021b). However, we posit that these models, while valuable, are not im-

[2]We demonstrate that estimating the conditional dynamic Fama coefficients is equivalent to adopting an instrumental variable approach akin to Barnichon and Mesters (2020). Our method surpasses the performance of Barnichon and Mesters (2020) when the IVs are valid and the effects of shocks are transitory.

[3]The delayed overshooting puzzle represents one of the extensively investigated conditional moment puzzles in the literature, as evidenced by studies such as Bacchetta and Van Wincoop (2021) and Müller et al. (2021). However, there is a notable gap in the literature concerning the establishment of empirical facts regarding other crucial moments conditional on shocks.

perative for explaining exchange rate puzzles. Considering that these puzzles can be contributed by multiple shocks, we advocate for further research aimed at aligning empirical (un)conditional moments with their (un)conditional counterparts respectively in theoretical models. The analytical tools developed in this paper can be instrumental in undertaking such investigations.

The remainder of the paper is structured as follows. Section 2 sets out the basic SVAR framework, explains the estimation approach, and outlines the identification strategy. Section 3 describes the data set used in estimation and presents the conditional behavior of exchange rate dynamics, including the shock contributions, the impulse responses and the structural scenario analysis. Section 4 documents the evidence of the conditional and unconditional UIP. Section 5 delves into a discussion of the theoretical implications stemming from our empirical findings regarding UIP dynamics. Section 6 provides extensive robustness checks regarding model specifications, identification schemes, and alternative estimation methods. Overall, our main results from the benchmark model are robust with these variations. Section 7 concludes.

2 Empirical strategy

Our starting point is an SVAR model of the form:

$$y_t = \sum_{i=1}^{K} B_i y_{t-i} + u_t, \tag{1}$$

where B_i denotes the coefficient matrix, u_t corresponds to the regression residuals with the variance-covariance matrix $Var(u_t) = \Sigma$, and y_t is a vector that contains endogenous variables measured at the quarterly frequency: US TFP, US real gross domestic product (GDP), the US short-term interest rate, US real stock prices, nominal exchange rates, foreign GDP, and the foreign interest rate. The nominal exchange rate is expressed as the US dollar price of foreign currency so that a decrease in the exchange rate indicates dollar appreciation. We take an aggregate of G7 excluding the US as the foreign country and employ a lag of four quarters in our benchmark model.

The model is estimated with Bayesian methods using an uninformative conjugate normal-inverse Wishart prior to infer the joint distribution of the parameters $(B_1, B_2, ..., B_K, \Sigma)$. The identification of the structural shocks amounts to searching for a mapping, A, between the regression residuals, u_t, and the structural shocks, ϵ_t, so that $u_t = A\epsilon_t$. By definition, matrix A, also known as the impact matrix, must satisfy $\Sigma = E(A\epsilon_t\epsilon_t' A') = AA'$. However, as is well known in the literature, these equations alone are insufficient to solve a unique impact matrix. As a result, auxiliary restrictions on matrix A are necessary to identify the structural shocks. The goal of our empirical exercise is to establish a unified framework to encompass four competing views that the literature has proposed to explain exchange rate movements. With this goal in mind, we jointly identify anticipated technology shocks, monetary shocks, safe asset demand shocks, and persistent interest rate shocks by imposing restrictions on impulse responses, the forecast error variance decompositions, and the historical decompositions during selected periods.

We proceed through three sequential steps in the following order. In the first step, we identify anticipated technology shocks, assuming that technology remains exogenous in the long run. In the second step, we identify monetary and safe asset demand shocks conditional on technology shocks, with the assumption that they have limited effects on technology in the long term. In the third step, we identify persistent interest rate shocks — those with lasting effects on interest rates but orthogonal to shocks identified in prior steps. This final step ensures that the identification of this shock does not preclude the other three shocks from exerting long-term impacts on interest rates.

2.1 Anticipated technology shocks

We follow the method proposed by Kurmann and Sims (2021) to identify anticipated technology shocks, which are shocks that have long-run effects on total factor productivity (TFP). To do so, we construct a linear combination of reduced-form innovations that maximizes its contribution to the variance of TFP forecast errors at a long horizon.[4] Specifically, suppose the SVAR model can be expressed in a moving average representation:

$$y_t = \sum_{i=0}^{\infty} C_i u_{t-i} = \sum_{i=0}^{\infty} C_i A \epsilon_{t-i}, \tag{2}$$

where C_i is the moving average coefficient matrix. Anticipated technology shocks can be identified by maximizing its contribution to the forecast error variance of TFP at horizon H:

$$a_j = \arg\max \frac{e_k' \left(\sum_{\tau=0}^{H} C_\tau a_j a_j' C_\tau' \right) e_k}{e_k' \left(\sum_{\tau=0}^{H} C_\tau \Sigma C_\tau' \right) e_k}, \tag{3}$$

where e_k is the selection vector with one in the k-th place and zeros elsewhere and a_j corresponds to the j-th column of the impact matrix A. Following Kurmann and Sims (2021), H is set as 80 in our benchmark model. Without loss of generality, TFP is ordered the first in the vector of endogenous variables, y_t, and the anticipated technology shock is ordered last in the shock vector, ϵ_t, (i.e., $k = 1$ and $j = 7$) to facilitate introducing other shocks in the following steps.

2.2 Monetary shocks and safe asset demand shocks

We identify monetary shocks and safe asset demand shocks by imposing traditional sign restrictions on the impulse responses and narrative sign restrictions on the structural shocks and the historical decompositions. For both shocks, we only impose sign restrictions for the quarter when the shock occurs. We remain agnostic about how the

[4] Another prevalent approach for identifying anticipated technology shocks is the Barsky and Sims (2011) method, which assumes that such shocks do not affect TFP on impact but contribute to most of its future variations. However, recent studies by Cascaldi-Garcia (2017) and Kurmann and Sims (2021) suggest that the effects of the Barsky and Sims (2011) anticipated technology shocks are not robust to data revision because the widely-used Fernald (2014) TFP series may suffer from cyclical mismeasurement errors of the true technology level. The Kurmann and Sims (2021) approach is devised to address this issue. Nevertheless, we find that the results from our analysis are mostly robust when we identify anticipated technology shocks using the Barsky and Sims method.

shocks may propagate beyond the current quarter and do not prohibit situations where the general equilibrium effects may bring about sign reversals in subsequent quarters. In line with most open-economy structural models, we assume that a contractionary US monetary shock may lead to a decrease in domestic output and stock prices, an appreciation of the dollar exchange rate, and an increase in the domestic interest rate.

The safe assets are typically appraised as a reliable store of value that contains almost no uncertainty about future payments. As the safe assets provide liquidity service and relax investors financial constraints, they are highly demanded during the episodes of global financial instability. Jiang et al. (2021) demonstrate that safe asset demand shocks explain substantial variances in the dollar exchange rate. In light of this, safe asset demand shocks are financial shocks that would raise the demand for liquidity services, push up the wedge between foreign yields and US treasury yields, and appreciate dollar exchange rates as the US treasury is the world's provider of the safe assets. Accordingly, we identify safe asset demand shocks by imposing the sign restrictions that a stronger safe asset demand causes a decrease in domestic interest rates and stock prices, and a dollar appreciation as predicted by the structural model in Chen (2021).

Even with sign restrictions on impulse responses, the task of achieving an economically plausible identification of safe asset demand shocks and monetary shocks is not trivial. The main challenge is that their effects can be similar to the dynamics elicited by other demand-type shocks (e.g., consumers preference shocks and government spending shocks). In addition, as argued by Antolín-Díaz and Rubio-Ramírez (2018), traditional sign restrictions on impulse responses do not guarantee that the implied shock realizations are consistent with the narrative account of key historical episodes. In consideration of the vulnerability of traditional sign restrictions approach, we attempt to improve the inference of SVAR by complementing traditional sign restrictions on impulse responses with a set of narrative sign restrictions that restrict the signs of realized shocks and the historical decompositions to be compatible with the established narrative account of key historical episodes following the approach of Antolín-Díaz and Rubio-

Ramírez (2018) and Giacomini et al. (2021).

Although there is a lack of agreement in the literature upon the entire time series of monetary shocks, it is possible to find some events upon which there is an agreement that important monetary policy shocks happened by a majority of researchers. For instance, Antolín-Díaz and Rubio-Ramírez (2018) make an important contribution by establishing a collection of most compelling and uncontroversial monetary shock episodes by cross-checking the Romer and Romer (1989) chronology, the updated Romer and Romer (2004) Greenbook residual series, the Gürkaynak et al. (2005) monetary shock series, and the FOMC meeting transcripts. Accordingly, we restrict the signs of monetary shock realizations to be consistent with the monetary policy chronology documented in Antolín-Díaz and Rubio-Ramírez (2018). We additionally assume that the stance of monetary policy was expansionary in 2008Q4 as the Fed cut the Federal Funds rate to zero for the first time in the history which is supported by the conventional monetary policy factor derived in Swanson (2021). Considering that the Volcker reform in 1979Q4 was the most apparent instance of an exogenous monetary shock in the postwar period, we further assume that monetary shocks make larger contributions to domestic short-term interest rates than other shocks during this period.[5]

For safe asset demand shocks, we impose narrative sign restrictions during specific episodes characterized by stock market crashes that triggered surges in the demand for safe assets. The selected episodes include "Black Friday" in 1987Q4, the bursting of the dot-com bubble in 2000Q4, the Global Financial Crisis in 2008Q3 and 2008Q4, the Euro Sovereign Debt Crisis of 2011Q3, and the Brexit referendum of 2016Q2. Figure 1 illustrates the borrowing rate, as defined by Cesa-Bianchi and Sokol (2022) as the sum of a 10-year government bond rate and the corporate bond spread from Gilchrist and Zakrajšek (2012). This rate is closely tied to shifts in financial conditions. Additionally,

[5] Antolín-Díaz and Rubio-Ramírez (2018) established eight monetary policy episodes in their chronology. Of the eight periods, April 1974 is not in our sample period. December 1990 episode is also excluded considering that the easing monetary policy can be a response to the Gulf War. Another reason for excluding this episode is that the policy change was announced on Dec 18, 1990 towards the end of the year.

we display the treasury basis, as defined by Jiang et al. (2021) as the yield gap between U.S. government bonds and currency-hedged foreign government bonds, which serves as a key indicator of the demand for U.S. dollar-denominated safe assets. Notably, most of the selected periods align with peaks in the borrowing rate and troughs in the treasury basis. We further assume that safe asset demand shocks contribute more to stock prices than other shocks in 2008Q4 when financial conditions deteriorated drastically as indicated by the Romer and Romer (2017) global financial distress series, the Piffer and Podstawski (2018) high-frequency gold prices, and the Cesa-Bianchi and Sokol (2022) borrowing rate series.

Narrative Sign Restriction 1: Monetary shocks are contractionary in 1979Q4, 1988Q4, and 1994Q1, and are expansionary in 1998Q4, 2001Q2, 2002Q4, and 2008Q4. For 1979Q4, the monetary shock is the most important contributor to the movements of domestic interest rates so that the absolute value of the contribution of the monetary shock is larger than that of any other structural shock.

Narrative Sign Restriction 2: Safe asset demand shocks are stronger in 1987Q4, 2000Q4, 2008Q3, 2008Q4, 2011Q3, and 2016Q2. For 2008Q4, the safe asset demand shock is the most important contributor to the movements of stock prices so that the absolute value of the contribution of the safe asset demand shock is larger than that of any other structural shock.

2.3 Persistent interest rate shocks

In the final step, we identify the shocks that trigger persistent shifts in interest rates, such as exogenous factors that may affect the inflation rate target and the natural rate of interest. It is noteworthy that our identification does not prohibit the shocks identified in the preceding steps from inducing long-lasting interest rate effects since previous papers allege that these shocks may contribute to natural interest rate movements (e.g., Laubach and Williams 2003 for anticipated technology shocks, Beaudry et al. 2022 for monetary shocks, and Del Negro et al. 2019 for safe asset demand shocks). The persistent interest rate shock that we attempt to identify is to encompass other factors that may cause secular shifts in interest rates, such as foreign factors (Wynne and Zhang 2018), demographic factors (Eggertsson et al. 2019), and wealth inequality (Mian et al. 2021). Recent papers (e.g., Müller et al. 2021, Zhang et al. 2021, and Schmitt-Grohé and Uribe 2022) attest that exogenous shifts in the natural interest rate can contribute to exchange rates substantially in open-economy dynamic stochastic general equilibrium (DSGE) models.

We identify persistent interest rate shocks by maximizing their contributions to interest rate variations at the 80-th horizon conditional on the shocks identified in previous steps. Different from TFP, the interest rate is an endogenous variable that can be contributed by a battery of shocks. When other shocks also contribute to the targeted endogenous variable significantly, the variance-maximizing approach may misidentify the shock of interest as demonstrated in Francis and Kindberg-Hanlon (2022). In this scenario, Francis and Kindberg-Hanlon (2022) recommend ameliorating the variance-maximizing approach by imposing additional sign restrictions. In this spirit, we restrict that persistent interest rate shocks raise interest rates steadily for the first 30 quarters.

To sum up, building on the merits of several state-of-the-art SVAR identification approaches, we identify the four structural shocks based on a comprehensive set of generally agreed assumptions imposed on impulse responses, variance decompositions, shock realizations, and historical decompositions. Our identification strategy is implemented with the following algorithm:

Algorithm 1 1. *We draw* $(\mathbf{B}, \mathbf{\Sigma})$ *from the normal-inverse-Wishart posterior distribution.*

2. *For each draw of the reduced-form parameters, we identify anticipated technology shocks with the maximal forecast error variance decomposition approach and obtain a candidate draw of impact matrix A. The last column of matrix A is determined by equation 3.*

3. *Rotate matrix A with an auxiliary matrix Q^* in the form of:*

$$Q^* = \begin{bmatrix} Q_{6\times 6} & 0_{6*1} \\ 0_{1*6} & 1 \end{bmatrix}, \tag{4}$$

by drawing 6×6 orthonormal matrix $Q_{6\times 6}$. Apply the new impact matrix $\tilde{A} = A * Q^*$ and keep the draws that satisfy both traditional and narrative sign restrictions for monetary and safe asset demand shocks. The design of Q^* matrix guarantees that the last column of matrix A is fixed when rotating the impact matrix.

4. *Repeat the above steps until 5000 draws have been obtained. For each draw of $\tilde{A}$ matrix, we identify the persistent interest rate shock by solving the third column of $\tilde{A}$ to maximize its contribution to US interest rate variations unexplained by the other identified shocks. We check whether the restriction that this shock raises US interest rates for the first 30 quarters is satisfied or not. We only retain the candidate draws of $\tilde{A}$ matrices that satisfy the sign restrictions.*[6]

3 Shock contributions and effects

Our benchmark model is estimated using quarterly data from 1975Q1 to 2018Q2. We follow the literature and utilize the Fernald (2014) quarterly TFP series, which corrects for latent factor utilization by exploiting first-order conditions from a firm's optimization problem. We use the real GDP as the output measure and the one-year treasury bond rate as the interest rate indicator. The one-year treasury rate allows us to compute one-year excess returns for assessing the UIP condition. It is also less binded by the zero lower bound than the official interest rate. However, we test the robustness of our results with the official monetary policy rate and a variety of other interest rates. The real stock prices are measured by the Standard & Poor's 500 composite index deflated by the core

[6] In our benchmark model, 782 draws are kept to capture the posterior distributions of the parameters. Francis and Kindberg-Hanlon (2022) combine the variance-maximization approach with the traditional sign restriction method in a different way. They maximize the forecast error variance conditional on the sign restrictions. When applying their approach to identify persistent interest rate shocks, the shocks induce similar effects to our benchmark results but explain little interest rate variations.

Consumer Price Index (CPI). Following Nam and Wang (2015), we take an aggregate of the G6 countries as the foreign country and aggregate the bilateral nominal exchange rate and foreign variables with the real GDP weights. All data, except for interest rate data, are taken in log-levels (multiplied by 100 to express the impulse response functions in percentage rates of variations). The details of our data construction and data sources are relegated to the Online Appendix.

We estimate the benchmark model with 4 lags. A prerequisite for the standard SVAR identification approach is that the information set contained in the SVAR model spans that of the agents so that the structural shocks can be recovered.[7] We perform the fundamentalness test proposed by Forni and Gambetti (2014), which confirms that our model contains sufficient information to recover the structural shocks as reported in the Online Appendix. In this section, we begin by comparing the contributions of the four identified shocks to provide insights into their relative importance. Next, we present the impulse responses to discuss the effects of the identified shocks, with a particular focus on exchange rates. Additionally, we investigate how the structural shocks transmit through the interest rate channel based on the structural scenario analysis of Antolin-Diaz et al. (2021) and Breitenlechner et al. (2022).

3.1 The contributions of the structural shocks

Figure 2 reports the posterior means along with the 68% and 90% confidence intervals for each shock's contribution to the endogenous variables at various horizons. The mean estimates suggest that the four shocks collectively account for approximately 85% of the variation in the US variables and around 75% of the volatility in exchange rates and foreign variables at a 30-quarter horizon. Given that the identified shocks explain most of the variation in all endogenous variables, our partial identification strategy does

[7]The standard SVAR identification method assumes that the structural shocks can be expressed as the linear combinations of the residuals of the linear projection of a vector of variables onto their past values. Chahrour and Jurado (2022) and Plagborg-Møller and Wolf (2022) relax this assumption to some extent by showing that the shocks can still be identified (but with more sophisticate procedure) when they are spanned by current, past, and future values of the observed macro variables.

not leave out any essential shocks, at least in the long run.

Our central evidence stems from the forecast error variance decomposition of the exchange rate, which shows that exchange rate dynamics are not dominated by a single type of shocks. Anticipated technology shocks and persistent interest rate shocks, respectively, explain 31% and 21% of the variations at the 30th quarter.[8] Safe asset demand shocks contribute moderately to exchange rate movements. They are responsible for 16% of immediate exchange rate dynamics, which becomes milder as forecast horizons extend. This stands in contrast to the findings of Itskhoki and Mukhin (2021a), who posit that financial shocks play a dominant role in accounting for exchange rate variations. The fundamental deviation arises from the fact that the success of Itskhoki and Mukhin (2021a)'s model hinges on the assumption of highly persistent financial shocks, a premise that our empirical results, as demonstrated in the next subsection, do not support. However, consistent with Itskhoki and Mukhin (2021a), we find that monetary shocks explain a small share, around 10%, of the exchange rate fluctuations.

The forecast error variance decompositions for other variables are standard, but they exhibit several noteworthy features. First, anticipated technology shocks are committed as the primary driver of long-run adjustments in TFP and GDP, as previously found in Kurmann and Sims (2021), and stock prices, as shown in Beaudry and Portier (2006). Second, both monetary shocks and persistent interest rate shocks are held accountable for domestic interest rate movements, together contributing to over 70% of the variations on impact. Compared to monetary shocks, persistent interest rate shocks explain more variations in both domestic and foreign output. Third, safe asset demand shocks explain a greater share of financial variables than macroeconomic variables in both domestic and foreign economies, which aligns with our definition of safe asset demand shocks as global financial shocks that push up the demand for liquidity services.

To evaluate the historical significance of various shocks to exchange rate fluctua-

[8] We also find that anticipated technology shocks are accountable for around 30% of real exchange rate dynamics when nominal exchange rates are replaced with real exchange rates in the benchmark model. This echoes corresponding estimates in Nam and Wang (2015) and Klein and Linnemann (2021).

tions, we present the historical decompositions for nominal exchange rates in Figure 3. The variance not explained by the four identified shocks is grouped into "other shocks". An inspection of Figure 3 suggests tha monetary shocks and persistent interest rate shocks make substantial contributions to exchange rates during the mid-1980s. In contrast, anticipated technology shocks were the primary driver of exchange rate fluctuations in the 1990s and early 2000s. This coincides with the development of information technology and the sharp increase in the economic value of patents (Cascaldi-Garcia and Vukotić 2022). We find a minor role of safe asset demand shocks in exchange rate volatility prior to the year of 2000. However, its role has increased since then and makes the most pronounced contribution to the dollar appreciation in 2008. This corroborates the idea proposed by Chen (2021) and Jiang et al. (2021) that exchange rate dynamics are primarily driven by safe asset demand shocks during the global financial crisis. In summary, exchange rate dynamics originate from the combined influences of all four identified shocks.

3.2 The effects of the structural shocks

The response of endogenous variables to one standard deviation of shocks is illustrated in Figure 4, which displays the median responses with 68% and 90% confidence limits shaded in dark and light areas, respectively. The figure clearly shows that exchange rates respond to the structural shocks in distinct patterns and magnitudes. Specifically, in the aftermath of a favorable anticipated technology shock, we observe an exchange rate appreciation, resembling a horizontal J-curve with the maximal impact occurring approximately two years after the shock. Additionally, anticipated technology shocks have strong expansionary effects on domestic GDP and stock prices, which is consistent with the notion of technology-driven economic boom. However, technology shocks have negligible effects on both domestic and foreign interest rates, as shown in Piffer and Podstawski (2018).

Following a monetary policy tightening, US interest rate shoots up by roughly 50 basis points immediately, which levels off after two years. This has the effect of dampening US GDP, devaluing US stock prices, and hiking foreign interest rates. Exchange rates appreciate persistently in an S-shape, with the peak response cropping up over three years after the shock. The causal evidence derived from our model lends support to a delayed overshooting of exchange rates. Furthermore, our research shows that US monetary shocks have moderately expansionary effects on foreign output, which may occur due to the depreciation of foreign currency against the US dollar.[9]

Safe asset demand shocks induce small and short-lived effects on both domestic and foreign output. An increase in the demand for safe assets, such as treasury securities, may reflect an elevated uncertainty in the global financial market, which triggers an immediate and sharp decline in stock prices. Both domestic and foreign interest rates decline as central banks systematically respond to the deteriorating economic and financial conditions. When investors endogenously fly to safety and pour capital into the US market, the dollar exchange rate appreciates in a hump-shaped pattern. It is worth noting that exchange rates overshoot immediately, with the largest response occurring within a quarter after the shock. The fresh empirical evidence that we add to the existing literature is that the exchange rate effects of safe asset demand shocks are more transient than predicted by the general equilibrium model of Chen (2021) and Eichenbaum et al. (2021).

Persistent interest rate shocks lead to sustained increase in US interest rates for more than 30 quarters, which unsurprisingly depresses the stock market, triggers foreign economic recessions, and appreciates dollar exchange rates. The exchange rate responses are persistent and resemble an L-shape. Surprisingly, the shock elicits transient expansionary effects on domestic GDP, which is contrary to the conventional wisdom that an interest rate hike may depress the aggregate demand of the economy.[10] Furthermore,

[9] As discussed in Iacoviello and Navarro (2019), models of international interest rate transmission typically suggest that the US monetary policy spills over to foreign economies through various channels, including the trade channel, the capital flow channel and the exchange rate channel. However, our results evince that the exchange rate channel dominates, as it implies expansionary spillover effects, while the other channels suggest the opposite effects.

[10] One way to rationalize this empirical finding is to appeal to the imperfect information models. Eco-

foreign interest rate responses are muted to persistent interest rate shocks, which intimates that the interest rate spillovers can be perplexing and can occur through divergent channels. On the one hand, the foreign central bank needs to lower its policy interest rate when confronting weaker GDP growth. On the other hand, the fear of capital outflow may impose upward pressure on foreign interest rates. The opposite effects through various channels may counteract each other.

In order to better understand the effects of identified shocks through the domestic interest rate channel, we conduct a structural scenario analysis of Antolin-Diaz et al. (2021) and Breitenlechner et al. (2022) to "shut down" the channel. The scenario analysis involves forecasting the effects of a shock conditional on the scenario when another offset shock leads to a certain path of the targeted variable. Specifically, we investigate the effects of anticipated technology shocks, safe asset demand shocks, and persistent interest rate shocks conditional on a sequence of US monetary policy shocks that counteract the effects on the domestic interest rates. The results of this policy counterfactual are presented as red dashed lines in Figure 4.

Upon comparing the structural scenario analysis with the impulse responses, we observe that the interest rate channel plays a more critical role in transmitting persistent interest rate shocks than other shocks. In the absence of the domestic interest rate channel, persistent interest rate shocks exhibit negligible effects on domestic variables and exchange rates, but have significant and long-lasting effects on foreign variables. This may be due to exogenous factors that persistently shift interest rates originating from the foreign economy, which lowers foreign natural rates of interest but raises the US rate.[11]

nomic agents need to learn what factors contribute to the increase in domestic interest rates over time. Initially, agents may interpret the persistent interest rate hike as a signal of strong aggregate consumption demand, as suggested by the Euler Equation, and expand their expenditure accordingly. However, as additional economic information is released, they soon learn the true economic state and cease expanding their expenditure once they recognize that the factor enhancing the interest rate level is not strong consumption demand.

[11]For example, foreign output growth shocks is a possible candidate for such a shock. In a simple two-country New Keynesian open-economy framework of Clarida et al. (2002), the domestic natural rate of interest, $\bar{r}r_t$,is determined by both the domestic potential output growth expectation, $E_t \Delta \bar{y}_{t+1}$, and the foreign output growth expectation $E_t \Delta y^*_{t+1}$. As formalized in equation 51 of Clarida et al. (2002), $\bar{r}r_t = \sigma_0 E_t \Delta \bar{y}_{t+1} + \kappa_0 E_t \Delta y^*_{t+1}$, where $\sigma_0 = \sigma - \gamma(\sigma - 1)$, $\kappa_0 = \gamma(\sigma - 1)$, $\sigma > 0$ denotes the inverse of the

When the domestic interest rate is allowed to respond, the spill-back effects of higher US rates on foreign country counteract the direct effects of the factor on foreign interest rates, as displayed in the structural scenario analysis. This leads to less significant total effects on foreign variables, as shown by the impulse responses (in solid lines).

4 Uncovered interest parity

One central hypothesis in most international models of exchange rate determination is the UIP condition. The UIP condition postulates that there is no arbitrage in the international asset market, which means that the expected returns on default-free assets should be equalized across countries. However, extensive literature has shown that this key condition may not hold in the data. In particular, countries with higher interest rates tend to have higher expected returns, resulting in a substantial share of excess returns being predictable. Consequently, investors can benefit from investing the higher interest rate currency, as the interest differential is magnified by an exchange rate appreciation. This empirical regularity is known as the "UIP puzzle".

Engel (2016) shows that the correlation between foreign exchange excess returns and the cross-country interest rate differentials reverses sign at long horizons, with contemporaneous high interest rate differentials predicting negative excess returns at horizons from four to seven years. In this section, we seek to provide further empirical evidence to explore the UIP puzzle and its sign reversal anomaly. To achieve this, we first implement the procedure developed by Engel (2016) to test unconditional UIP within our SVAR model. We then investigate the UIP conditional on identified shocks and examine whether the UIP violation depend on the shocks at play. Finally, we illustrate through a behavioral New Keynesian open-economy model that the inherent characteristic of shock-dependent UIP failure may help explain the observed occurrence of unconditional UIP reversal.

intertemporal substitution elasticity, and $0 < \gamma < 1$ represents the share of home spending on foreign countries. It's easy to verify that σ_0 is always positive, while κ_0 can be negative when $\sigma < 1$.

4.1 Unconditional UIP

To investigate the UIP condition, we define the one-year excess return from investing in the foreign bond from period t to the year after (period $t+4$) in terms of the US currency as:

$$\rho_{t,4} \equiv (i_t^* - i_t) + (s_{t+4} - s_t), \tag{5}$$

where $(i_t^* - i_t)$ is the foreign less domestic interest rate differential and $(s_{t+4} - s_t)$ is the year-to-year change in the logarithm of the nominal exchange rate.[12] Following the seminal work of Fama (1984), a vast literature tests the unconditional UIP by considering the famous Fama regression of the foreign exchange excess return on the interest differential:

$$\rho_{t,4} = \alpha + \beta(i_t^* - i_t) + e_{t,4}, \tag{6}$$

where $\alpha = 0$ and $\beta = 0$ if the UIP condition holds.

The standard UIP test of equation 6 can be extended to an arbitrary h-period ahead horizon. Particularly, if $E_t\rho_{t,4} = 0$ for any t, it follows that $E_t\rho_{t+h,4} = 0$ for any h when applying the law of iterated expectations. Therefore, the future one-year excess return is unpredictable at all horizons under the UIP condition. We test this condition with a series of regressions in a manner similar to the local projection in Jordà (2005):

$$\rho_{t+h,4} = \alpha_h + \beta_h(i_t^* - i_t) + e_{t+h,4}, \tag{7}$$

where h is non-negative with the initial Fama (1984) equation being the special case of $h = 0$.

The upper left portion of panel (a) in Figure 5 displays our median estimates of β_h for $h = 0, 1, 2, ...30$ along with their 68% and 90% heteroscedastic-and-autocorrelation

[12] We also consider the annualized one-quarter excess return, $\rho_{t,1} \equiv (i_t^* - i_t) + 4(s_{t+1} - s_t)$, and find that the results reported in this section are robust to this alternative definition.

consistent corrected confidence limits using Newey-West standard errors. The median estimates of β_h reveal that the excess return on the high-interest currency is predicted to increase at short horizons, but then switch direction at longer horizons. Specifically, the estimates plummet from 2.4 on impact to -1.8 at the 30th quarter, indicating the reversal of the UIP puzzle. The plotted 90% confidence interval confirms that the UIP puzzle and its reversal are statistically significant.

4.2 Conditional UIP

4.2.1 Impulse responses of foreign exchange excess returns

While the failure of unconditional UIP generally emerges as a consensus in the existing literature, economists hold differing opinions on the conditional UIP in response to the structural shocks.[13] In this subsection, we investigate whether UIP holds conditional on each of the four identified shocks. Following Eichenbaum and Evans (1995), the existent literature assesses conditional UIP by inspecting the impulse responses of the excess returns as formulated by:

$$\frac{\partial \rho_{t+h,4}}{\partial \epsilon_t^s} = \frac{\partial i_{t+h}^*}{\partial \epsilon_t^s} - \frac{\partial i_{t+h}}{\partial \epsilon_t^s} + \frac{\partial s_{t+h+4}}{\partial \epsilon_t^s} - \frac{\partial s_{t+h}}{\partial \epsilon_t^s}, \tag{8}$$

where ϵ_t^s represents the structural shock that the excess return responses are conditional on.

Figure 6 presents in the top row the median impulse responses of one-year excess returns, along with the 68% and 90% confidence sets. It is shown that only persistent interest rate shocks entail significantly positive excess return responses at short horizons. We further decompose the excess return responses into the responses of cross-country interest differential, $\partial(i_{t+h}^* - i_{t+h})/\partial\epsilon_t^s$, and the responses of the exchange rate difference,

[13] For instance, Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) argue that there is a conditional UIP violation following a monetary shock, while Kim and Roubini (2000), Bjørnland (2009) and Rüth (2020) detect no UIP violation for the same shock. Kim et al. (2017) suggest that "the UIP assumption significantly fails during the Volcker era", but "tends to hold in the post-Volcker era".

$\partial(s_{t+h+4} - s_{t+h})/\partial\epsilon_t^s$. Figure 6 clearly manifests that the responses of the excess return are primarily driven by the responses of the exchange rate difference. Even when our estimation bears witness to significant interest differential responses, the excess return responses are insignificant because the uncertainty around the exchange rate difference is enormous. This calls into question the validity of the conventional conditional UIP test based on the excess return responses obtained from Equation 8. In particular, a significantly negative interest rate differential combined with an insignificantly negative exchange rate difference may not necessarily deliver a significantly negative excess return. The substantial uncertainty associated with the exchange rate difference may dominate and deteriorate the power of the conventional impulse-response-based conditional UIP test.

The conventional conditional UIP test also has two other limitations. First, the test based on the excess return responses is conceptually incongruent with the unconditional UIP test, which relies on the estimation of Fama regression coefficients. The Fama regression tests whether the excess returns can be predicted by cross-country interest rate differentials, whereas the excess return responses evaluate whether a current shock can modify the expectation of future excess returns. Thus, the results from the conditional UIP test shed little insight into the unconditional UIP violation. Second, the excess return responses only allow for the investigation of UIP conditional on a single shock. As far as we know, the SVAR literature has not attempted to analyze UIP conditional on multiple shocks simultaneously, which may aid in assessing the model's overall efficacy in elucidating the UIP puzzle.[14]

[14] Assessing UIP conditional on multiple shocks can be conveniently implemented within the framework of DSGE models by setting the variance of uninterested shocks to zero. Consequently, employing an empirical approach to evaluate UIP conditional on multiple SVAR shocks can facilitate comparisons with theoretical models.

4.2.2 Conditional Fama regression

This paper proposes a new approach for assessing the conditional UIP. The method involves estimating dynamic Fama regression models in a counterfactual scenario where only the shock that the UIP is conditional on is allowed, while all other shocks are set to zero.[15] To explain the idea in detail, we express the moving average form of the SVAR model as:

$$y_t = \sum_{m=0}^{\infty} \Theta_m \epsilon_{t-m}, \tag{9}$$

where ϵ_{t-m} refers to the vector of the structural shocks, and $\Theta_m = C_m A$ represents the structural moving average matrix coefficients. To evaluate the UIP conditional on the k-th shock, we construct the counterfactual data of the j-th endogenous variable by using the following equation:

$$y_t^{(j,k)} = \sum_{m=0}^{t-1} \Theta_m^{(j,k)} \epsilon_{t-m}^{(k)} + d_t^j, \tag{10}$$

where $\epsilon_{t-m}^{(k)}$ denotes the k-th structural shock in ϵ_{t-m}, $\Theta_m^{(j,k)}$ is the j-th row and k-th column of Θ_m, and d_t^j represents deterministic terms. We test the conditional UIP by estimating dynamic Fama regression equations:

$$\rho_{t+h,4}^{(k)} = \alpha_h^{(k)} + \beta_h^{(k)} (i_t^{*(k)} - i_t^{(k)}) + e_{t+h,4}^{(k)}, \tag{11}$$

[15] This idea has found applications in assessing conditional dynamic Fama coefficients in DSGE models by simply setting certain shock variance to zero as in Valchev (2020) and Itskhoki and Mukhin (2021a). In the SVAR literature, Cormun and De Leo (2022) and Chahrour et al. (2023) employ a similar approach to evaluate the contemporaneous Fama coefficients of β_0. We were not aware of both papers during the initial circulation of our draft and express gratitude to Pierre De Leo for bringing it to our attention. It's noteworthy that, unlike our study, both papers do not evaluate dynamic Fama coefficients, nor do they explore the relationship between conditional and unconditional dynamic Fama coefficients as well as the instrumental approach of Barnichon and Mesters (2020) as we do. The evaluation of counterfactual data by Chahrour et al. (2023) may differ from our approach, given that their model does not assume invertibility. In their model, endogenous variables also depend on the realization of future shocks.

where $\rho^{(k)}_{t+h,4} = (i^{*(k)}_{t+h} - i^{(k)}_{t+h}) + (s^{(k)}_{t+h+4} - s^{(k)}_{t+h})$. [16]

The essence of estimating equation 11 is equivalent to estimating the relationship between ρ_{t+h} and $i^*_t - i_t$ using an IV approach with ϵ^k_1, ϵ^k_2, ..., ϵ^k_{t+h+4} as IVs simultaneously. In the first stage, the fitted values of $\rho_{t+h,4}$ and $i_t - i_t$ are obtained from equation 10, while the estimation of equation 11 constitutes the second-stage estimation. The IV is valid only if the residual of Equation 11 is uncorrelated with the conditioned shocks. Barnichon and Mesters (2020) also propose using lag sequences of SVAR identified shocks as instruments to estimate structural equations.[17] However, as explained in Appendix B, the approach proposed by Barnichon and Mesters (2020) relies on a single lag of the shock to derive moment conditions at a time and thus may encounter the issue of weak identification when the effects of shocks on explanatory variables are transient. In this scenario, our method outperforms Barnichon and Mesters (2020) in terms of efficiency as all lags of structural shocks are utilized simultaneously.

The conditional dynamic Fama regression is not limited to testing the UIP conditional on a single shock. We can easily extend the exercise to multiple shocks. For instance, to evaluate whether UIP holds conditional on the first two shocks , we can derive the counterfactual data $y^{(j,1,2)}_t = \sum_{m=0}^{t-1} \Theta^{(j,1)}_m \epsilon^{(1)}_{t-m} + \sum_{m=0}^{t-1} \Theta^{(j,2)}_m \epsilon^{(2)}_{t-m} + d^j_t$ and apply it to the conditional Fama regression model. In an extreme case, the Fama regressions conditional on all the seven shocks together are tantamount to the respective unconditional Fama regressions, given that the summation of the historical decompositions recovers the data itself. Furthermore, in the next section, we will demonstrate that un-

[16] Similarly, we can assess conditional ex ante Fama coefficients through the estimation of $\tilde{E}_t\rho^{(k)}_{t+h,4} = \alpha^{(k)}_h + \tilde{\beta}^{(k)}_h(i^{*(k)}_t - i^{(k)}_t) + e^{(k)}_{t+h,4}$ where $\tilde{E}_t\rho^{(k)}_{t+h,4} = (\tilde{E}_t i^{*(k)}_{t+h} - \tilde{E}_t i^{(k)}_{t+h}) + (\tilde{E}_t s^{(k)}_{t+h+4} - \tilde{E}_t s^{(k)}_{t+h})$. We construct the model-implied expectation of the j-th endogenous variable conditional on the k-th shock using $\tilde{E}_t y^{(j,k)}_{t+h} = \sum_{m=h}^{t+h-1} \Theta^{(j,k)}_m \epsilon^{(k)}_{t+h-m} + d^j_{t+h}$. The detailed results are available upon request.

[17] When contrasting Equation 11 with Barnichon and Mesters (2020), we interpret the Fama equations as structural rather than reduced-form equations. In a standard model where UIP holds, all shocks qualify as valid IVs. However, in recent literature that underscores endogenous deviations from UIP, as seen in Itskhoki and Mukhin (2021b) and Devereux et al. (2023), the validity of IVs depends on whether the conditioned SVAR shocks are independent from the shocks driving the deviation from UIP in the theoretical model.

conditional Fama coefficients can be expressed as weighted averages of conditional Fama coefficients, where the weights are determined by the contributions of shocks to interest rate differentials. Hence, the conditional UIP tests based on counterfactual data and conditional forecasts are conceptually compatible with the unconditional UIP tests.

Figure 5 displays the Fama regression estimates conditional on each of the four shocks individually and the four shocks together, which provides several noteworthy observations. First, the Fama coefficients conditional on all four shocks together exhibit a similar pattern as the unconditional Fama coefficients. This implies that the four identified shocks, when considered together, effectively encapsulate the empirical relationship between foreign exchange excess returns and the interest rate differential witnessed in the data.[18]

Second, the dynamic Fama coefficients are shock dependent at short horizons, which exhibit distinctive patterns contingent on the specific shocks under consideration. Among the four identified shocks, the rejection of UIP only occurs in response to monetary and persistent interest rate shocks, with higher interest rates triggering a concurrent upsurge in excess returns. This contrasts with the results of the impulse-response-based UIP test, where we cannot reject UIP conditional on monetary shocks. Henceforth, our test relying on conditional Fama coefficients may yield different conclusions from the one based on the impulse response of excess returns.

Third, the conditional Fama coefficients are not significant for any of the four shocks at long horizons so that the empirical findings do not provide adequate evidence to support the presence of conditional UIP reversal. Recent literature abounds with theoretical models that evinces the sign reversal of UIP conditions across time horizons. Some economists reconcile this phenomenon with models featuring deviation from full information rational expectation (FIRE) models, including Candian and De Leo (2023), Kolasa

[18] Conditioning the estimated Fama coefficients on all four shocks simultaneously results in narrower confidence bands compared to conditioning on a single shock. This reduction in band width is primarily attributed to the uncertainty surrounding shock identification. The collective identification of four shocks introduces less uncertainty than identifying each shock individually, especially when the identified shocks together explain most of the variations in endogenous variables.

et al. (2022), and Na and Xie (2022). Other scholarly investigations have proffered supplementary mechanisms, such as convenience yields (Valchev 2020) and infrequent portfolio adjustment (Bacchetta and Van Wincoop 2010 and Bacchetta et al. 2023). Despite the diversity in the elucidated mechanisms, it is noteworthy that prevailing theoretical explanations share a foundational commonality: the UIP reverses its sign conditional on at least one shock, which is responsible for the sign reversal of unconditional UIP. [19] Considering that both the existing literature and our empirical examination are silent about UIP reversal at long horizons, a scholarly imperative emerges to explore alternative theoretical frameworks capable of generating unconditional UIP reversal without reliance on conditional UIP reversals.

5 Discussion

In this section, we formally investigate the conditions to construct a theoretical model that demonstrates unconditional UIP reversal, yet refrains from exhibiting such reversal conditional on any exogenous shocks. We aim to establish a set of necessary and sufficient conditions for unconditional UIP reversal through the presentation of a proposition and two lemmas.

Proposition 1: For any open-economy model that has an moving average representation (including SVAR model, international real business cycle model and New Keynesian open economy model), the relationship between the unconditional dynamic Fama coefficients, denoted as β_h, and the Fama coefficients conditional on the k-th shock, expressed as $\beta_h^{(k)}$, is as follows:

[19] The only exception we identify is Engel (2016). While presenting a partial equilibrium model where the reversal is solely unconditional and results from the convolution of monetary and liquidity shocks, it lacks a comprehensive discussion on the conditions for unconditional UIP to manifest, not only in theoretical models but also in empirical models.

$$\beta_h = \sum_{k=1}^{K} \beta_h^{(k)} \frac{Var(i_t^* - i_t|\epsilon_t^k)}{Var(i_t^* - i_t)}, \tag{12}$$

where $Var(i_t^* - i_t|\epsilon_t^k)$ represents the variance of the cross-country interest rate differential conditional on the k-th shock, ϵ_t^k, while $Var(i_t^* - i_t)$ represents the unconditional variance with $Var(i_t^* - i_t) = \sum_{k=1}^{K} Var(i_t^* - i_t|\epsilon_t^k)$. As the forecast horizon, h, approaches infinity, both β_h and $\beta_h^{(k)}$ converge to zero.

The proof of Proposition 1 is detailed in Appendix C, which also contains the explicit form of Fama coefficients for DSGE models with both full-information rational expectations and models with distorted beliefs. Proposition 1 asserts that the unconditional Fama coefficients can be defined as a weighted average of conditional Fama coefficients, with the weights determined by the shocks' contributions to the variance of current interest rate differential.[20]

This proposition sheds light on empirical evidence from previous studies. Firstly, studies such as Itskhoki and Mukhin (2021a) and Cormun and De Leo (2022) underscore the significance of conditioned shocks in explaining exchange rate dynamics, particularly when the dynamics of conditional UIP mirror the coefficients of unconditional UIP. For example, Itskhoki and Mukhin (2021a) contend that "if financial shocks play a crucial role in the exchange rate dynamics, the model replicates a negative unconditional Fama coefficient." Proposition 1 challenges this perspective by demonstrating that the dynamics of conditional UIP may take precedence when conditioned shocks are pivotal

[20]While Proposition 1 is employed to elucidate UIP violations, its relevance extends beyond the realm of International Economics and can be applied to interpret the estimation of structural macro equations. An illustrative example is found in the research of Carvalho et al. (2021), where it is argued that the bias in ordinary least squares estimates (OLSE) of the Taylor rule is proportionate to the fraction of regressor variance attributed to monetary shocks. This point can be illuminated using Proposition 1 as well. As elucidated in the preceding section, conditional coefficients function as an IV estimator, proving consistent and converging to the true value when conditioned shocks act as valid instrumental variables. In the context of Taylor rule estimates, all shocks are valid IVs except for the monetary shock itself. Therefore, Proposition 1 implies that the large sample bias of OLSE is the discrepancy between the true parameter value and the coefficient conditional on monetary shocks, rescaled by the contribution of monetary shocks to the regressor.

in explaining interest rate dynamics rather than movements in exchange rates. Secondly, Engel et al. (2022) contend that the relationship between interest rate differentials and foreign exchange returns is time-varying. Proposition 1 implies that this variability in parameters can emerge when the Fama coefficients are estimated to be shock-dependent, and the contributions of shocks to interest rate differentials vary over time.

Most importantly, Proposition 1 has the potential to provide a generalized approach to explaining unconditional UIP reversals. In previous theoretical studies, when concentrating on a single shock, unconditional UIP reversal is achieved by constraining this shock to exert a dominant influence on interest rate dynamics, necessitating conditional reversal in $\beta_h^{(k)}$ to bring about β_h reversal. In the subsequent discussion, we illustrate that the pattern in conditional Fama coefficients can deviate from the unconditional Fama coefficients within a multi-shock framework. Proposition 1 suggests that the unconditional Fama coefficients, β_h, are expected to fall within the range defined by $Min(\beta_h^{(1)}, \beta_h^{(2)}, ..., \beta_h^{(K)})$ and $Max(\beta_h^{(1)}, \beta_h^{(2)}, ..., \beta_h^{(K)})$. In light of this insight, Lemma 1 establishes a collection of necessary conditions that must be met to engender unconditional UIP reversal.

Lemma 1 Unconditional UIP reversal ($\beta_0 > 0$ and $\exists h_0 > 0$ so that $\beta_{h_0} < 0$) can be attained only if:

1. There exists at least one shock, ϵ_t^a that predicts a positive contemporaneous relationship between interest rate differential and foreign exchange excess returns (i.e., $\beta_0^a > 0$);

2. There exists at least one shock, ϵ_t^b, and a forecast horizon $h_0 > 0$ that predicts a negative relationship between interest rate differential and foreign exchange excess returns at this horizon ((i.e., $\exists\ h_0 > 0$, $\beta_{h_0}^b < 0$).

Lemma 1 demonstrates that unconditional UIP reversal is unattainable when all shocks consistently forecast identical directional movements between interest rate differentials and foreign exchange returns across all time horizons. Nevertheless, Proposition

1 also imply that conditional UIP reversals are not obligatory for the occurrence of unconditional UIP reversal, except when researchers specifically focus on a single shock and assign it a weight of 1. In this context, Lemma 2 furnishes a set of sufficient conditions that facilitates the unconditional UIP reversal.

Lemma 2 Unconditional UIP reversal ($\beta_0 > 0$ and $\exists h_0 > 0$ so that $\beta_{h_0} < 0$) can be attained in a DSGE model if researchers are allowed to set the variance of exogenous shocks and if:

1. There exists at least one shock, ϵ_t^n, which contributes to the variance of cross-country interest rate differential (i.e. $Var(i_t^* - i_t|\epsilon_t^n) \neq 0$) and predicts a negative relationship between interest rate differential and foreign exchange excess returns on impact (i.e., $\beta_0^n < 0$) and non-positive relationship for other horizons ($\beta_h^n \leq 0$ for $h > 0$).

2. There exists at least one shock, ϵ_t^p, which contributes to the variance of cross-country interest rate differential (i.e. $Var(i_t^* - i_t|\epsilon_t^p) \neq 0$) and predicts a positive relationship between interest rate differential and foreign exchange excess returns on impact (i.e., $\beta_0^p > 0$) and non-negative relationship for other horizons ($\beta_h^p \geq 0$ for $h > 0$).

3. β_h^n converges to 0 more slowly than β_h^p.

The first two conditions specify that the Fama coefficients conditional on ϵ_t^n and ϵ_t^p have opposite signs, yet they do not exhibit reversals. Researchers can readily achieve a positive β_0 by imposing a substantial shock variance for ϵ_t^p. Condition 3 ensures the existence of a horizon $h_0 > 0$ where β_{h_0} is negative, even in the presence of a large shock variance for ϵ_t^p, as $\beta_{h_0}^p$ becomes arbitrarily small, and $\beta_{h_0}^n$ is not.

The prior literature has often explained unconditional UIP reversal by achieving it conditionally upon a specific shock. This has been accomplished through the incorporation of intricately designed mechanisms and the imposition of restricted model parameters. The significance of Lemma 2 lies in its provision of alternative perspectives for generating unconditional UIP reversals.[21] In this vein, the conditions outlined in previous studies may not be essential for eliciting unconditional UIP reversals. To illustrate this point, we present examples using the models from Valchev (2020) and Candian and De Leo (2023). It is important to note that we are not asserting the invalidity of mechanisms introduced in prior papers. Rather, our argument posits that the conditions in previous models can be eased if conditional UIP reversal is not imperative for the generation of unconditional UIP reversal.

5.1 Application 1: UIP Reversal under Passive Monetary Policy in Valchev (2020) Model

Valchev (2020) devises a model incorporating convenience yields that can induce UIP reversal under the conditions of an active monetary policy and a passive fiscal policy, particularly when a sufficiently persistent tax rule is in effect.[22] The rationale behind this lies in the notion that, under these specific circumstances, the dynamics of the system are governed by complex roots. Consequently, the impact of monetary policy shocks on government debt, and thus convenience yields and excess returns, reverse signs as forecast horizons extend. However, it should be noted that the presence of complex roots results in impulse responses of almost all endogenous variables to all shocks reversing their signs (as shown in the Online Appendix), which may not align with empirical observations. Hence, the creation of UIP reversal comes at the expense of

[21] Lemma 2 highlights that unconditional UIP reversals in DSGE models do not necessitate the appearance of conditional UIP reversals for all shocks. In fact, it allows for reversals conditional on a subset of shocks in the model. In addition, while the necessary and sufficient conditions specified in Lemmas 1 and 2 apply to each draw in admissible sets of SVAR models, significance detection may be challenging, considering both estimation and identification uncertainty. In our empirical analysis, a notable share of posterior draws implies negative dynamic Fama coefficients conditional on safe asset demand and anticipated technology shocks, but these estimates are generally insignificant. Therefore, we are not contesting the validity of the previous mechanism but complementing the previous studies by proposing an alternative approach to generate unconditional UIP reversals.

[22] It is worth noting that Valchev (2020) defines foreign exchange excess returns in an opposing manner to our own definition.

compromising the model's performance in other dimensions.[23]

Furthermore, as demonstrated by Engel (2016), the phenomenon of UIP reversal is not confined to dollar exchange rates. It is observed in countries like Japan, where interest rates have remained close to zero, and monetary policy has been characterized as passive for nearly three decades. In this section, we illustrate that, in line with the principles outlined in Lemma 2, it is possible to induce UIP reversal in an environment characterized by passive monetary policy and active fiscal policy, all without necessitating the presence of complex roots within the equilibrium system.

In the analytical model of Valchev (2020), foreign exchange excess returns are linked to convenience yields, a factor reliant solely upon the debt level. When active fiscal policy and passive monetary policy are in effect, the real debt value aligns with the present value of primary surpluses, with monetary policy making adjustments passively to maintain the equilibrium path of prices and debt. As a result, the response of excess returns to monetary policy shocks is notably muted, primarily because the debt level, and consequently, convenience yields, remain unresponsive to such shocks.

Nonetheless, under the same model, UIP experiences deviations when conditioned on government spending shocks and tax shocks. Deficit fiscal shocks trigger an upsurge in inflation and, consequently, interest rates. Under passive monetary policy regime, the central bank's tolerance for inflation is high, leading to the erosion of debt value as the deficits are inflated away. This, in turn, increases convenience yields and foreign exchange excess returns. As exemplified in the upper panel of Figure 7, Fama coefficients remain subdued when conditioned on monetary shocks, but they take on negative values when conditioned on tax and government spending shocks, lacking any discernible reversals. Given that no shocks anticipate a positive relationship between interest rates differential and excess returns so that the necessary conditions outlined in Lemma 1 are not satisfied, unconditional UIP reversal cannot be delivered with the original Valchev

[23] The model under consideration is the analytical model outlined in Section 3 of Valchev (2020). It is noteworthy that the impacts on macro variables exhibit less significant sign reversal in the quantitative model detailed in Section 4 of Valchev (2020). We thank Rosen Valchev for pointing this out.

(2020) model under passive monetary policy.

In light of this failure, we adapt the Valchev (2020) model to enable foreign exchange excess returns to be positively influenced by domestic interest rates and thus engender a negative relationship between interest rate differentials and foreign exchange excess returns:

$$E_t \Delta s_{t+1} + (i^* - i_t) = (1/\xi - 1) i_t - \frac{\gamma \Psi}{\xi} b_{ht}, \tag{13}$$

We derive Equation 13 by assuming that domestic investors possess the capability to invest in foreign bonds, employing a leverage ratio denoted as ξ, where ξ is greater than 1. This assumption allows foreign exchange returns to respond to domestic interest rates with minimal deviation from the Valchev (2020) model, while keeping the other equilibrium conditions in the model unchanged.[24] Further elaboration of the model and the derivation of expected foreign exchange excess returns can be found in the Online Appendix.

The introduction of the modified excess return expression leads to the observation that a contractionary monetary shock results in positive foreign exchange excess returns and cross-country interest rate differentials. In the lower panel of Figure 7, it is evident that the Fama coefficients conditional on monetary shocks converge to zero at a quicker pace than those conditional on fiscal shocks, which aligns with the sufficient conditions in Lemma 2. Through appropriate parameterization of the shock variances, we can successfully induce UIP reversals under passive monetary policy. In summary, this example

[24]The original Valchev (2020) model corresponds to a situation where ξ is set to 1.Permitting leverage greater than 1 suggests that domestic households' investment in foreign assets involves elevated risk. This corresponds with the concept of asymmetric risk sharing as explored in Maggiori (2017), which results in a situation where the US balance sheet is marked by risky assets, contrasting with foreign financial institutions that predominantly hold safer assets. An alternative method to motivate Equation 13 involves introducing money and market bonds into households' decision-making process, as demonstrated in Candian and De Leo (2023) and Engel and Wu (2023), but with the assumption of complementarity between money and government bonds. However, this approach introduces additional endogenous variables, leading to a more significant departure from the original Valchev (2020) model.

demonstrates that, inspired by Lemma 2, UIP reversal can be achieved with a wider range of parameters and more general conditions.

5.2 Application 2: UIP Reversal with a Variant of Candian and De Leo (2023) Model

Candian and De Leo (2023) induce UIP reversal by extending an open-economy New Keynesian model with the incorporation of distorted beliefs, where both shock misperception and over-extrapolative beliefs are deemed essential. The model successfully generates UIP reversal, specifically conditional on the four shocks that they include in their model, including technological and preference shocks occurring in both the home and foreign countries. However, as in Valchev (2020), the mechanism that generates conditional UIP reversal also leads to impulse responses in multiple endogenous variables, such as GDP, inflation, and consumption, reversing their signs, which does not align with the prevailing empirical evidence.

We adapt the quantitative model initially presented by Candian and De Leo (2023), replacing their shocks with the four specific shocks under scrutiny in our SVAR model. In most cases, we have retained parameter values consistent with Candian and De Leo (2023). However, we have introduced a lower persistence for monetary policy shocks to distinguish them from persistent interest rate shocks. For a comprehensive understanding of the model's intricacies, including its description, solution methodology, and the calibration of model parameters, please refer to the Online Appendix. In Figure 8, it is evident that the Fama coefficients, conditioned upon anticipated technology shocks, exhibit negative values, whereas the coefficients conditioned upon the other three shocks demonstrate positivity. Unconditional UIP can be achieved by appropriately setting the shock variances.

Candian and De Leo (2023) demonstrates that both shock misperception and over-extrapolative beliefs are considered essential to generate the UIP reversal. Similar to

Candian and De Leo (2023), we cannot generate the UIP reversal under the assumption of full-information rational expectations, as the Fama coefficients conditional on each of the shocks exhibit the same sign. However, in the Online Appendix, we illustrate that our model does not necessitate both shock misperception and over-extrapolative beliefs. We can achieve unconditional UIP reversal with only one of the two belief distortion assumptions as conditional UIP reversal is not a prerequisite. Admittedly, there is a conspicuous disparity between the theoretical Fama coefficients and their counterparts derived from the SVAR analysis. Nevertheless, our findings provide a significant revelation: the occurrence of conditional UIP reversal does not stand as a necessary precondition for the manifestation of unconditional UIP reversal. Due to the insignificance observed in our estimates of conditional Fama coefficients, our empirical results do not provide a decisive resolution regarding the validity of our proposed mechanism, based on the convolution of multiple shocks, versus the prior mechanism relying on conditional UIP reversal. However, the discussion in this section generalizes the conditions necessary for generating UIP reversal, notably without inducing a corresponding reversal in the impulse responses of other endogenous variables.

6 Robustness checks

6.1 Identifying one shock at a time

In this subsection, we separately identify each of the four shocks one at a time. We employ the same identification restrictions as in our benchmark model to ensure consistency in our analysis. Our goal is to determine whether the separately identified shocks might confound with the endogenous responses to unidentified shocks.

Figures A.1 and A.2 of the Appendix display the impulse responses and forecast error variance decompositions, respectively. Our results suggest that the effects of the separately identified anticipated technology shocks and safe asset demand shocks are consistent with our benchmark results. However, both monetary shocks and persistent interest rate shocks elicit more persistently negative effects on TFP, GDP, and stock prices. In fact, persistent interest rate shocks and monetary shocks account for up to 34% and 16% of the long-run TFP volatility, respectively, indicating that both shocks may have been confounded with unfavorable anticipated technology shocks. This confounding problem can distort the inference of the effects on exchange rates. It is worth noting that the exchange rate responses are muted to monetary shocks, which can be explained by the fact that the dollar appreciation following ”true” monetary shocks is offset by the depreciation provoked by ”true” negative anticipated technology shocks.

6.2 Country pairs

We also investigate the bi-lateral exchange rate dynamics by re-estimating our SVAR model with data of the US and each of the G6 countries. The impulse responses of the bi-lateral exchange rates are largely compatible with our benchmark results as presented in the Online Appendix. However, there exists some heterogeneity across country pairs. In particular, we observe that monetary shocks and the persistent interest rate shocks induce insignificantly dollar depreciation responses for the UK. Moreover, safe asset demand shocks result in more significant dollar appreciation against the Great British pound and the Canadian dollar, indicating that financial investors may view the US dollar as safer assets against sterling and the Canadian dollar than against other currencies.

We also test the UIP conditions for country pairs with our conditional Fama regression. In most cases, the confidence bands of the conditional Fama coefficients become wider, making it difficult to reject the conditional UIP, as shown in the Online Appendix. However, Japan is an outlier that calls for special attentions. We generally cannot reject the UIP conditional on any shocks based on the responses of Yen excess returns. In contrast, we are able to reject the UIP conditional on all shocks at short horizons when delivering our verdicts based on the conditional dynamic Fama regression coefficients. This again underscores that the estimates of dynamic Fama coefficients offer an effective measure of testing conditional UIP.

6.3 Alternative identification strategies and other robustness checks

We also identify anticipated technology shocks using the patent-based IV proposed by Klein and Linnemann (2021), which is constructed as the unpredictable component of patent application growth by its own lags and the Philadelphia Fed's Survey of Professional Forecasters. We then identify the other three shocks using the same identification assumption as the benchmark model, conditional on the IV-identified anticipated technology shock. The effects and contributions of all shocks are largely robust, as detailed in the Online Appendix.

There is an growing literature that uses high-frequency asset price movements during a narrow window around monetary policy announcements to construct instrumental variables (IVs) for identifying monetary shocks (Gertler and Karadi 2015, Jarociński and Karadi 2020). When identifying monetary shocks with the high-frequency IVs of Jarociński and Karadi (2020) and Miranda-Agrippino and Ricco (2021), we find that the exchange rate responses largely align with our benchmark results as depicted in the Online Appendix. However, we observe that domestic output rises in response to tight monetary shocks, which may take shape because the monetary policy proxies are predictable by macroeconomic and financial indicators, as discussed in Bauer and Swanson (2022).

A burgeoning literature has emerged that uses IVs derived from intra-daily changes in gold prices around a collection of events to identify financial shocks (Piffer and Podstawski 2018 and Georgiadis et al. 2021). Considering that gold is the ultimate safe asset, we identify safe asset demand shocks using the IV proposed by Piffer and Podstawski (2018). As depicted in the Online Appendix, high-frequency identified safe asset demand shocks lead to a sharp and persistent decline in TFP and a depreciation of the dollar, which suggests that these shocks might contain information about technology factors.[25]

[25] We urge caution when applying this method to quarterly SVAR models. First, the IVs are observed at a higher than quarterly frequency, and there is no scientific methodology to aggregate them to a quarterly frequency. Second, many intra-quarter effects of high-frequency shocks might be misinterpreted by the

We also report a battery of other robustness checks in the Online Appendix, covering various aspects such as: (1) switching to alternative lag structure of the SVAR model (with the lag lengths ranging from 2 to 6 quarters), (2) replacing the nominal exchange rate with the real exchange rate, (3) substituting real GDP with consumption, investment, and hours, (4) swapping stock prices for the global risky asset factor of Miranda-Agrippino and Rey (2020), (5) examining various interest rates, including the official monetary policy rate, the three-month treasury rate, and the two-year treasury rate, instead of the one-year treasury bond rates, (6) utilizing only the sample by 2008Q4 before unconventional monetary policy era, and (7) adopting the Minnesota prior for our Bayesian estimation. Overall, the conclusions from the benchmark model remain unchanged with these variations, as reported in the Online Appendix.

7 Concluding remarks

While a substantial literature has probed the determinants of exchange rate dynamics, most studies isolate the causal effects of individual shocks, attributing exchange rate dynamics predominantly to a dominant shock. In contrast, our paper employs a comprehensive approach to jointly disentangle and identify four types of shocks — anticipated technology shocks, monetary shocks, safe asset demand shocks, and persistent interest rate shocks. We apply generally agreed-upon assumptions across various dimensions, including impulse responses, forecast error variance decompositions, and historical decompositions. Our findings reveal that exchange rates derive from the collective impacts of all four shocks, each influencing exchange rates in distinct patterns.

Furthermore, we evaluate UIP deviations both conditionally and unconditionally. In contrast to existing literature justifying unconditional UIP reversal through mechanisms generating reversals in UIP conditional on specific shocks, our empirical findings show

SVAR models as exogenous shocks. Third, the proxy SVAR approach does not guarantee that the signs of the shock realizations agree with the narrative account for key historical episodes when the IVs are sparse or subject to significant measurement errors.

no significant evidence of conditional UIP reversal alongside unconditional UIP reversal. Our mathematical proof demonstrates that unconditional UIP reversal can arise as a collective impact of different shocks, even without a reversal in UIP conditional on any single shock. Given that conditional UIP reversal is not a necessary prerequisite for unconditional UIP reversal, the conditions stipulated in earlier models to generate unconditional UIP can be non-obligatory. We illustrate this point using models proposed by Valchev (2020) and Candian and De Leo (2023).

Our paper contributes to the literature by establishing an empirical framework for evaluating conditional moments and examining the relationship between conditional and unconditional moments. However, an important and improvable task in understanding exchange rate dynamics is to align theoretical moments with not only unconditional moments but also their conditional counterparts. The current studies are limited in this regard, warranting further work on both empirical and theoretical fronts. On the empirical side, there is a need for documenting more robust stylized facts regarding the conditional relationship between exchange rates and other endogenous variables with refined identification methods. On the theoretical side, researchers may enhance models in order to align with the conditional moments uncovered in empirical studies. We believe this opens a promising avenue for future research, enhancing our understanding of the mechanisms determining exchange rate movements.

The Online Appendix is available at:

https://www.dropbox.com/sh/skxveo0t6bes8zm/AADhJEbvbgFcPKjAac3sgzMHa?dl=0

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Tables

Table 1: A SUMMARY OF THE TRADITIONAL SIGN RESTRICTIONS

	US GDP	US Interest Rate	Stock Price	Real Exchange Rate
Monetary	$-$	$+$	$-$	$-$
Safe Asset Demand		$-$	$-$	$-$
Persistent Interest		$+_{30}$		

Notes: A "+" ("−") sign indicates that the impulse response of the variable is restricted to be positive (negative) in the quarter the shock occurs. We stipulate that the persistent interest rate shock raises interest rate for 30 periods and denote this restriction as $+_{30}$ in the table.

Table 2: NARRATIVE SIGN RESTRICTIONS

Dates	Events	Shocks	Narrative Sign
1979Q4	Volcker Reform	Monetary	$+$
1988Q4	Start of a Tightening Cycle	Monetary	$+$
1994Q1	Start of a 12-month Tightening Cycle	Monetary	$+$
1998Q4	Greenspan Put (LTCM)	Monetary	$-$
2001Q2	Greenspan Put (dot-com bubble)	Monetary	$-$
2002Q4	50 bp Rate Cut	Monetary	$-$
2008Q4	Cut Rate to Zero	Monetary	$-$
1987Q4	Black Monday	Safe Asset Demand	$+$
2000Q4	dot com bubble bursts	Safe Asset Demand	$+$
2008Q3	Global Financial Crisis	Safe Asset Demand	$+$
2008Q4	Global Financial Crisis	Safe Asset Demand	$+$
2011Q3	European Sovereign Debt Crisis	Safe Asset Demand	$+$
2016Q2	BREXIT Referendum	Safe Asset Demand	$+$

Notes: A "+" ("−") sign indicates that the sign of the identified shock is restricted to be positive (negative) in the selected periods.

Figures

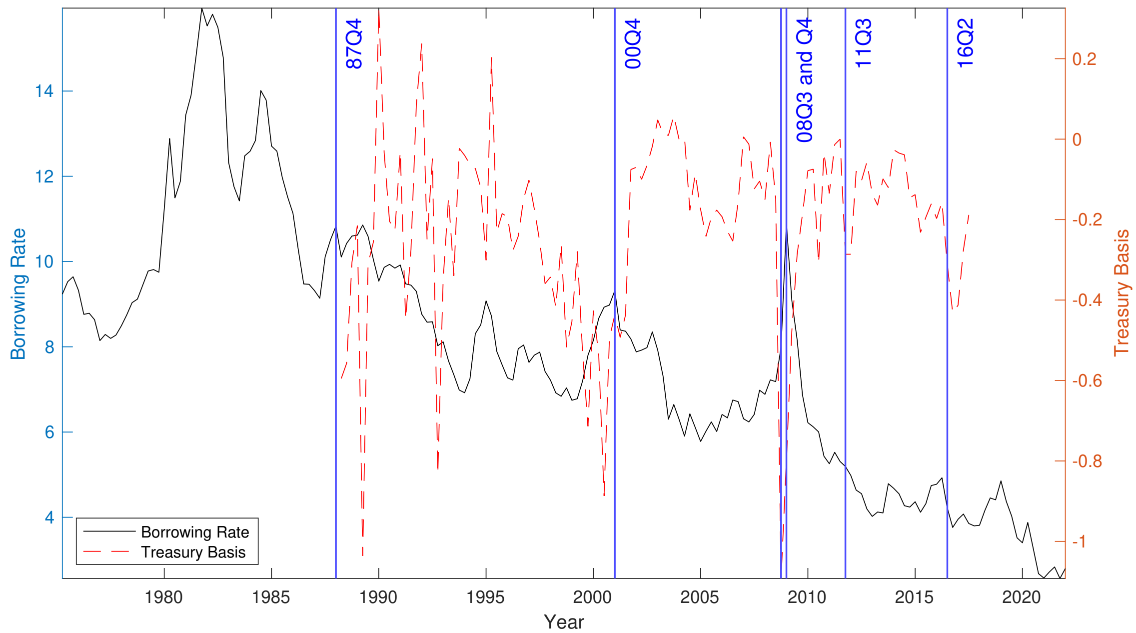

Figure 1: BORROWING RATES AND TREASURY BASIS

Notes: Borrowing rates are defined as the sum of Gilchrist and Zakrajšek (2012) credit spread and ten-year treasury bond rates as in Cesa-Bianchi and Sokol (2022). Treasury basis are from Jiang et al. (2021).

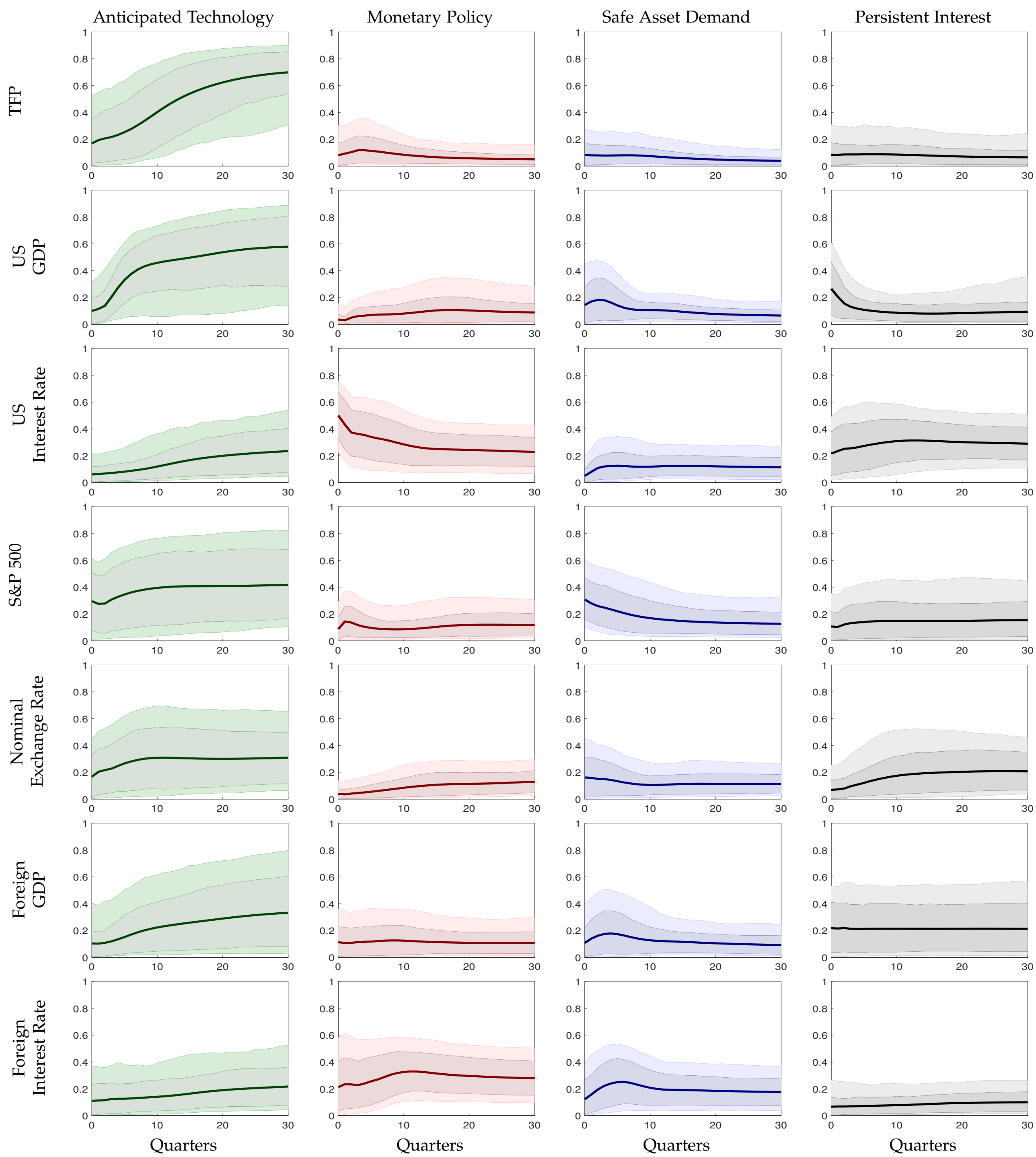

Figure 2: FORECAST ERROR VARIANCE DECOMPOSITIONS

Notes: The solid line and shaded areas report the mean, 68% and 90% confidence intervals.

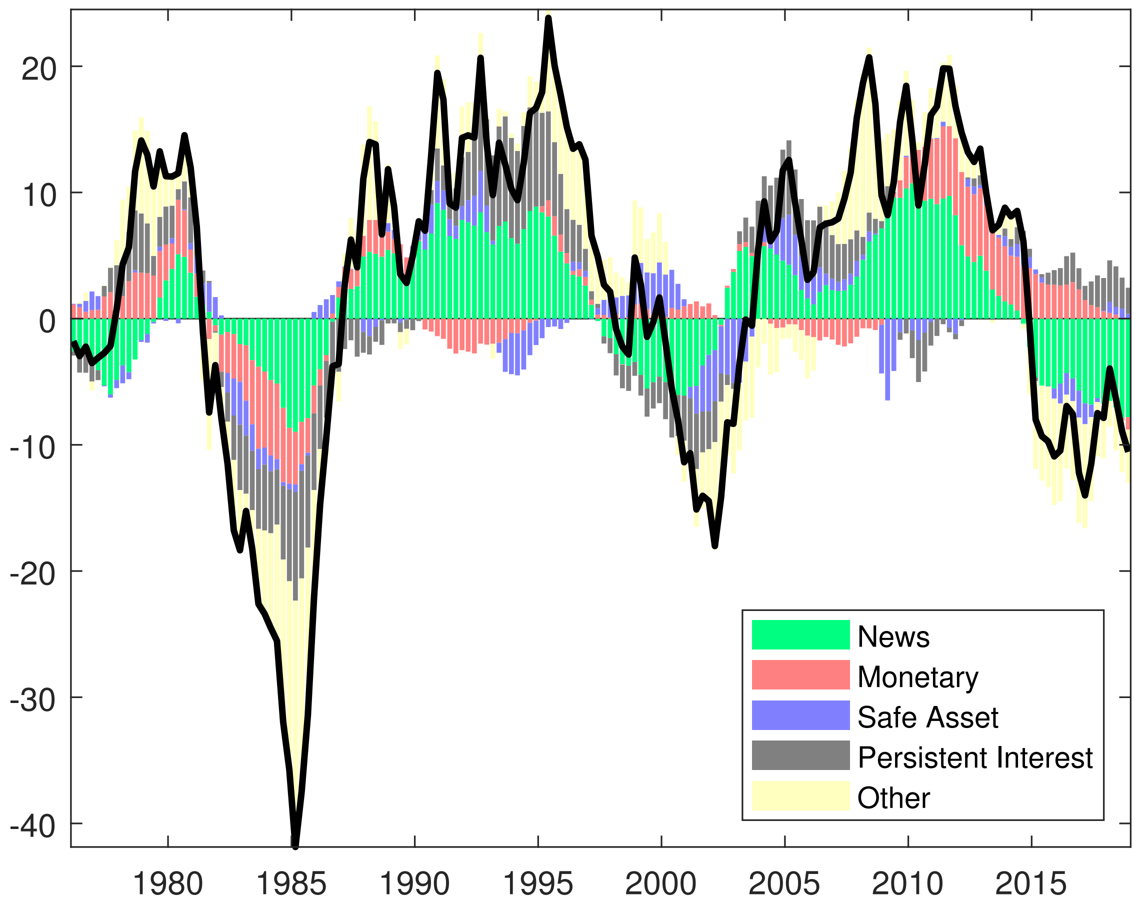

Figure 3: HISTORICAL DECOMPOSITIONS FOR NOMINAL EXCHANGE RATES

Notes: This figure shows the decomposition of historical data (black solid line) into components due to the four identified shocks. We report the average decomposition from the posterior distributions. The yellow shaded area represents the components due to the rest unidentified shocks.

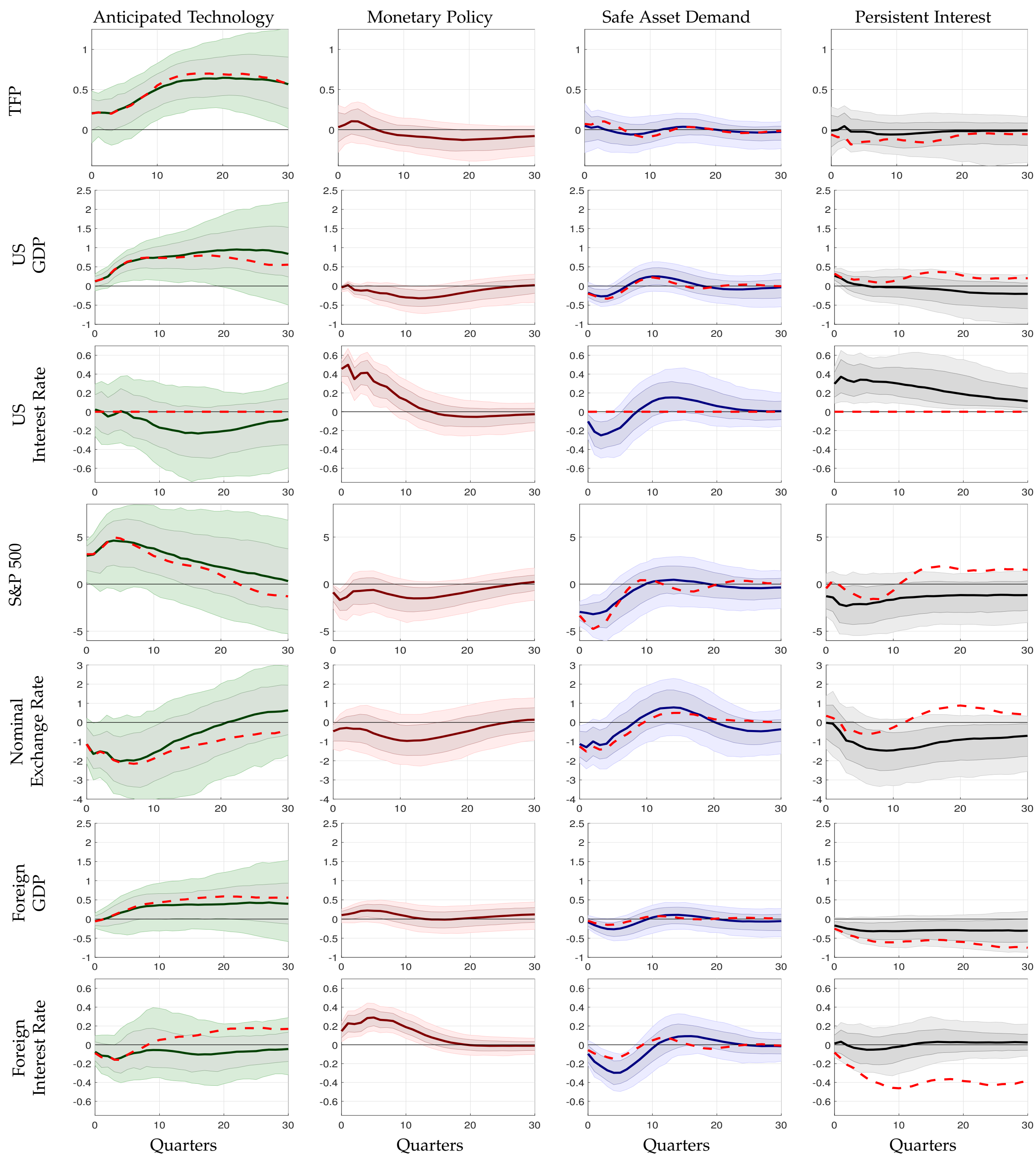

Figure 4: IMPULSE RESPONSES

Notes: This figure shows impulse responses to the four identified shocks in the benchmark model. The solid lines are the actual impulse responses. The shaded areas are the 68% and 90% confidence intervals. The red dashed lines report the structural scenario analysis when the domestic interest rate channel is shut down.

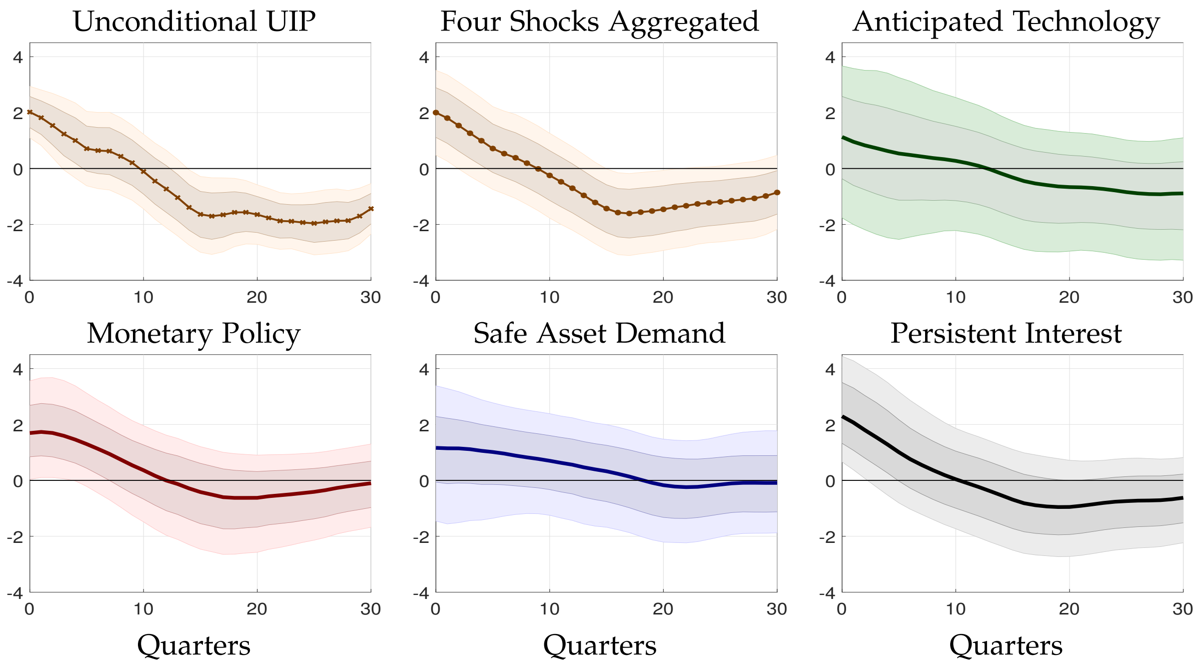

Figure 5: SLOPE COEFFICIENTS OF DYNAMIC FAMA REGRESSIONS

Notes: The figure shows the dynamic Fama coefficients estimated with data from 1975Q1 to 2018Q4. The shaded areas are the 90% and 68% confidence intervals.

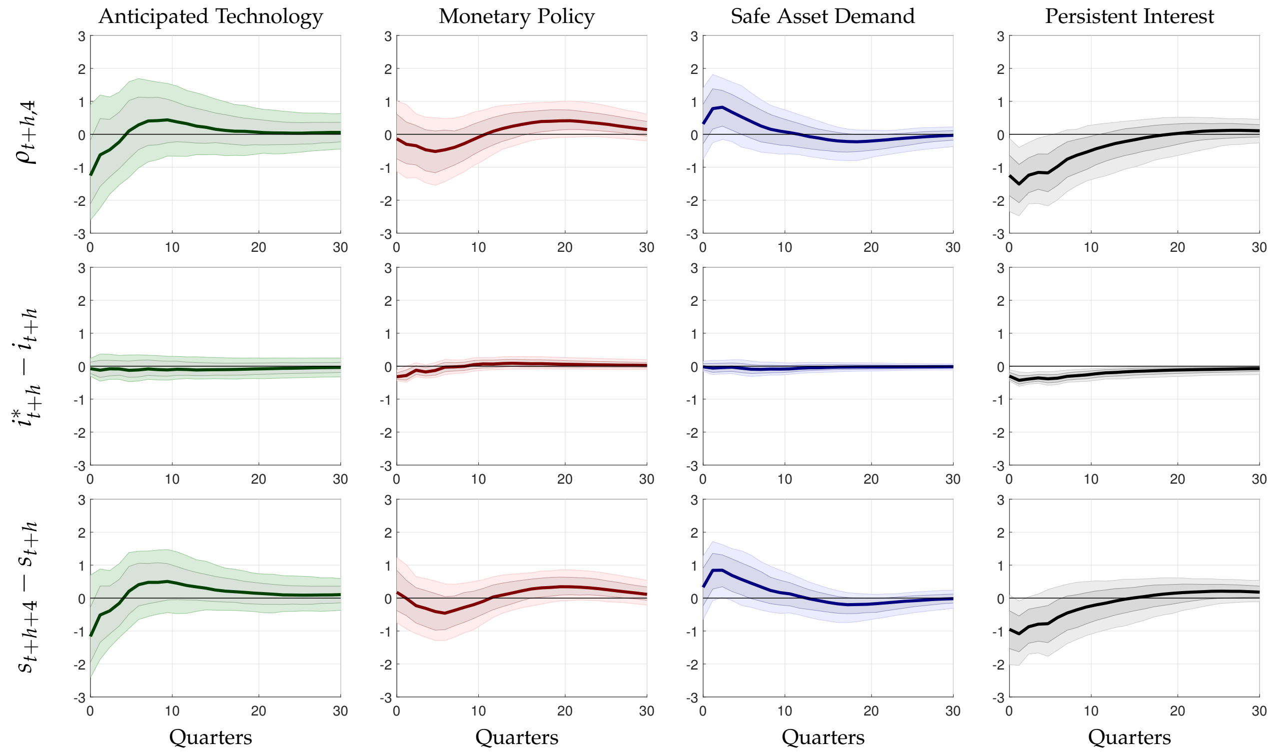

Figure 6: EXCESS RETURN RESPONSES

Notes: This figure shows impulse responses of 1-year excess return $\rho_{t+h,4}$, interest rate difference $i^*_{t+h} - i_{t+h}$ and exchange rate difference $s_{t+h+4} - s_{t+h}$ in the benchmark (1975Q1∼2018Q4) model, together with the 90% and 68% confidence intervals.

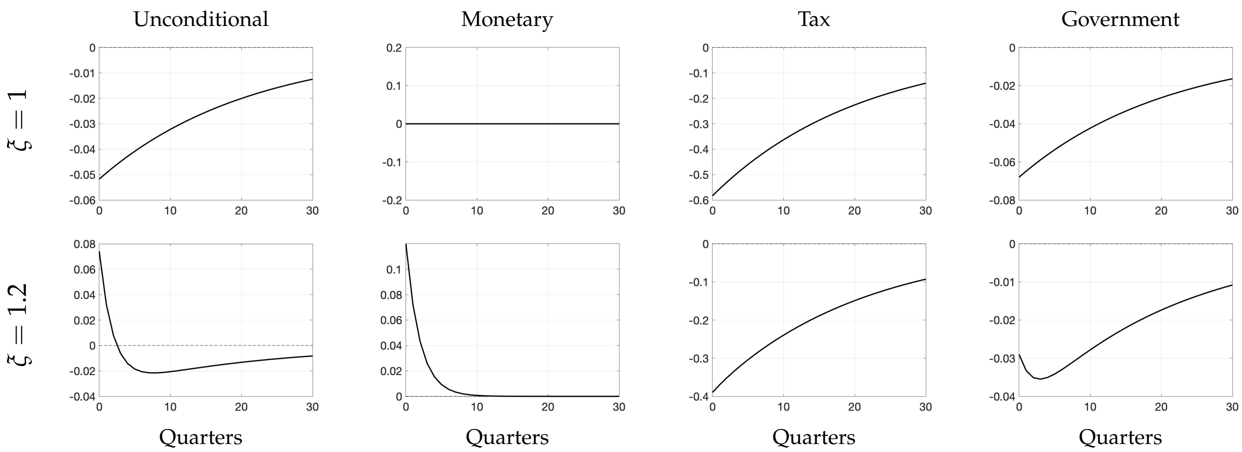

Figure 7: SLOPE COEFFICIENTS OF FAMA REGRESSIONS IN AN ANALYTICAL MODEL

Notes: The upper panel illustrates the Fama coefficients within the analytical model of Valchev (2020) under passive monetary and active fiscal policy. In contrast, the lower panel presents the Fama coefficients for the same model, accounting for the scenario where domestic investors have the capability to invest in foreign bonds with a leverage factor denoted as ξ.

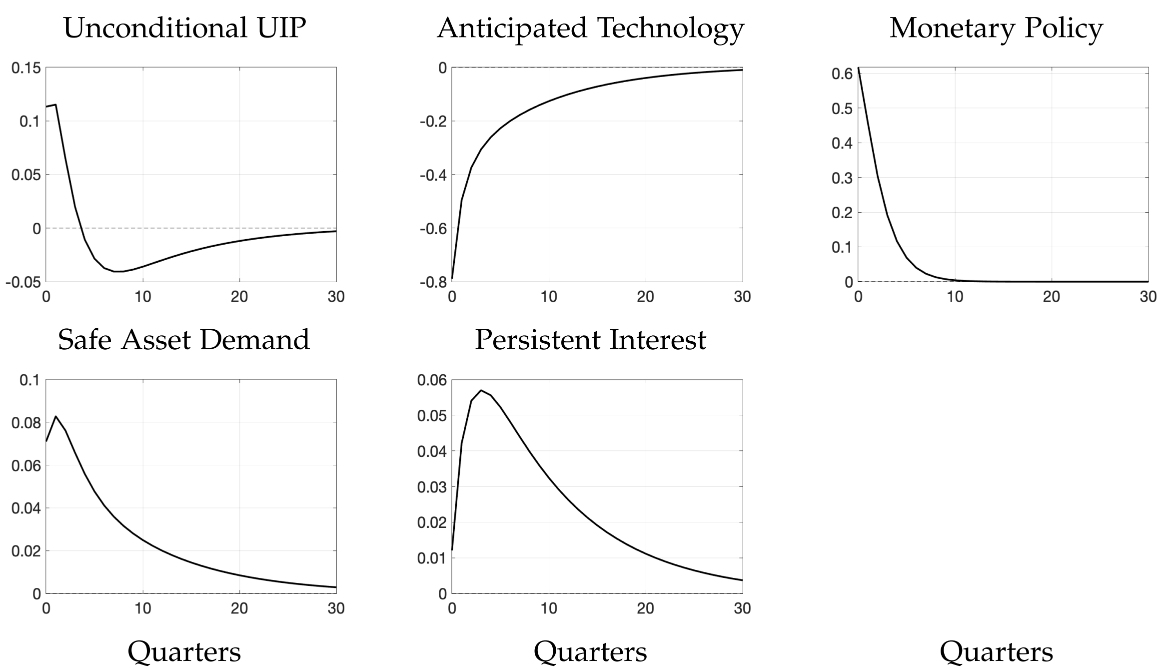

Figure 8: SLOPE COEFFICIENTS OF FAMA REGRESSIONS IN A QUANTITATIVE MODEL

Notes: The figure illustrates the dynamic Fama coefficients derived from a quantitative model that modifies Candian and De Leo (2023) framework by substituting their exogenous shocks with the shocks under examination in our SVAR model.

A Appendix

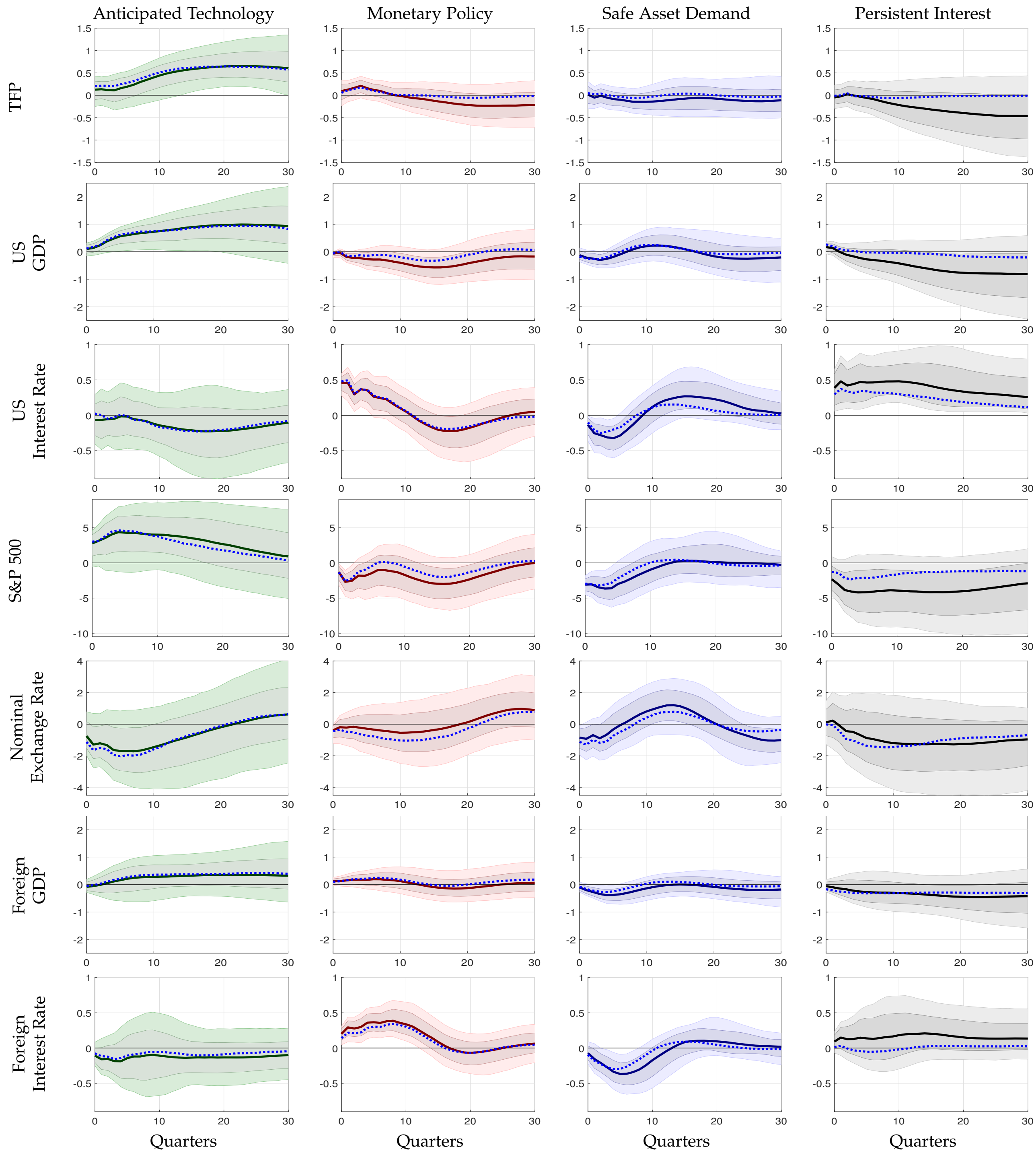

Figure A.1: IMPULSE RESPONSES: SINGLE IDENTIFIED SHOCK

Notes: This figure shows impulse responses when each of the shock is identified separately one at a time. The blue dashed line is the median impulse response of the joint identification from the benchmark model. The shaded areas are the 68% and 90% confidence intervals.

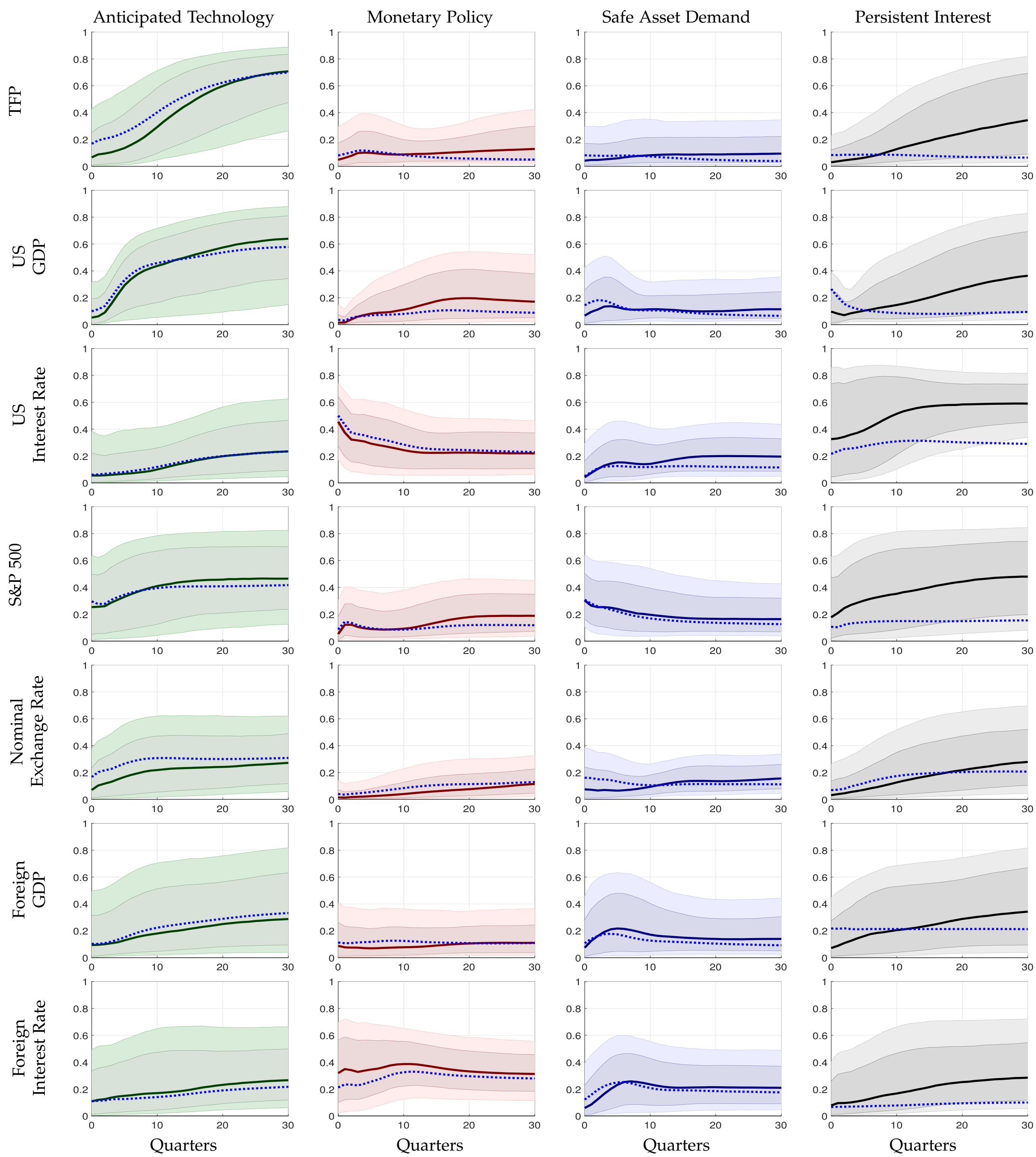

Figure A.2: FORECAST ERROR VARIANCE DECOMPOSITIONS: SINGLE IDENTIFIED SHOCK

Notes: This figure shows the forecast error variance decompositions when each of the shock is identified separately one at a time. The blue dashed line is the mean forecast error variance decompositions of the joint identification from the benchmark model. The shaded areas are the 68% and 90% confidence intervals.

B The IV approach of Barnichon and Mesters (2020)

Our paper proposes a new approach to test conditional UIP by estimating the dynamic Fama regressions of Engel (2016) with counterfactual data and conditional forecasts where only the shocks that the UIP is conditional on are active. In this appendix, we compare our approach with the IV approach of Barnichon and Mesters (2020).

Barnichon and Mesters (2020) attempt to estimate a structural time series equation of the form:

$$z_t = \gamma x_t + u_t, \tag{B.1}$$

where z_t and x_t are scalar endogenous variables, u_t is an error term, and γ is the structural parameter of interest. They propose that the endogeneity problem can be cracked by using SVAR shocks as IVs. The SVAR model should include both z_t and x_t, which, without loss of generality, are assumed as the first two variables in the model. Suppose the SVAR model has a structural moving average form:

$$\begin{bmatrix} z_t \\ x_t \\ o_t \end{bmatrix} = \sum_{m=0}^{\infty} \begin{bmatrix} \Theta_m^{(z,z)} & \Theta_m^{(z,x)} & \Theta_m^{(z,o)} \\ \Theta_m^{(x,z)} & \Theta_m^{(x,x)} & \Theta_m^{(x,o)} \\ \Theta_m^{(o,z)} & \Theta_m^{(o,x)} & \Theta_m^{(o,o)} \end{bmatrix} \begin{bmatrix} \varepsilon_{z,t-m} \\ \varepsilon_{x,t-m} \\ \varepsilon_{o,t-m} \end{bmatrix}, \tag{B.2}$$

where o_t is a vector of other endogenous variables. Barnichon and Mesters (2020) propose using the lag sequences of the structural shocks as IVs to estimate parameter γ in equation (B.1). For illustration purpose, we assume that $\epsilon_{x,t}$, $\epsilon_{x,t-1}$, ..., $\epsilon_{x,t-H}$ are valid instruments and multiplying them by both sides of equation (B.1):

$$z_t \epsilon_{x,t-m} = \gamma x_t \epsilon_{x,t-m} + u_t \epsilon_{x,t-m}, \qquad m = 0, \cdots, H. \tag{B.3}$$

Imposing the validity assumption of the IVs, $E(u_t \epsilon_{x,t-m}) = 0$, we take expectation for both sides of equation (B.3) and obtain the moment conditions:

$$\Theta_m^{(z,x)} = \gamma \Theta_m^{(x,x)}, \qquad m = 0, \cdots, H. \tag{B.4}$$

as $E(z_t \epsilon_{x,t-m}) = \Theta_m^{(z,x)}$ and $E(x_t \epsilon_{x,t-m}) = \Theta_m^{(x,x)}$. γ is solved by the above moment conditions described in equation (B.4) with the generalized method of moments. As a result, Barnichon and Mesters (2020) estimate the target coefficient with $H + 1$ moment conditions derived from utilizing one lag of $\epsilon_{x,t}$ as the instrument at a time. By contrast, we obtain the fitted value of z and x utilizing all lags of the structural shocks simultaneously. Moreover, Barnichon and Mesters (2020) estimator is inefficient when the effects of ϵ_x on x_t are transient (i.e., $\Theta_m^{(x,x)} = 0$) and thus suffer from the weak IV problem. The relevance of our IV is not dictated by the persistence of the shock effects. Instead, it depends on whether the shock is of historical significance to the explanatory variable x_t.

C Evaluating Dynamic Fama Coefficients in DSGE Models

In this appendix, we begin by presenting the proof of Proposition 1. We subsequently deduce the moving average representation of the DSGE model both with full information rational expectations and with distorted beliefs as presented in Candian and De Leo (2023), which can be used to calculate the Fama coefficients in DSGE models.

C.1 Proof of Proposition 1

Tbe moving average representation of a DSGE model:

$$y_t = \sum_{m=0}^{\infty} \Theta_m \epsilon_{t-m}, \tag{C.1}$$

where y_t is the endogenous vector, ϵ_t are structural shocks with a diagonal variance-covariance matrix $E\epsilon_t\epsilon_t^{'} = \Sigma_\epsilon$, and Θ_m are moving average coefficients, which converges to zero as m extends.The variance and auto-covariance matrix of the endogenous vector is characterized by:

$$\begin{aligned} Var\left(y_t\right) &= \sum_{m=0}^{\infty} \Theta_m \Sigma_w \Theta_m' \\ Cov\left(y_t, y_{t+h}\right) &= \sum_{m=0}^{\infty} \Theta_m \Sigma_w \Theta_{m+h}^{'} \end{aligned} \tag{C.2}$$

The variance and auto-covariance matrix of the endogenous vector conditional on the k-th shock are:

$$\begin{aligned} Var\left(y_t|\epsilon_t^k\right) &= \sigma_k^2 \sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_m' \\ Cov\left(y_t, y_{t+h}|\epsilon_t^k\right) &= \sigma_k^2 \sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_{m+h}^{'} \end{aligned} \tag{C.3}$$

where e_k is the selection (column) vector with one in the k-th place and zero elsewhere, and σ_k^2 is the variance of the k-th shock..

Without loss of generality, we include the cross-country interest rate differential, $i_t^* - i_t$, as the first variable in the endogenous vector, y_t, while $i_{t-1}^* - i_{t-1} + s_t - s_{t-1}$ the second endogenous variable. The ordinary least square estimation of unconditional Fama coefficients, β_h, can be expressed as:

$$\begin{aligned}\beta_h &= \frac{Cov(i_{t+h}^* - i_{t+h} + s_{t+h+1} - s_{t+h}, i_t^* - i_t)}{Var(i_t^* - i_t)} \\ &= \frac{e_1' Cov(y_t, y_{t+h+1}) e_2}{e_1' Var(y_t) e_1} \\ &= \frac{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_{m+h+1}') e_2}{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_m') e_1},\end{aligned} \tag{C.4}$$

The dynamic Fama coefficients conditional on the k-th shock, $\beta_h^{(k)}$, are formulated by

$$\begin{aligned}\beta_h^{(k)} &= \frac{e_1' Cov(y_t, y_{t+h+1} | \epsilon_t^k) e_2}{e_1' Var(y_t | \epsilon_t^k) e_1} \\ &= \frac{\sigma_k^2 e_1' (\sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_{m+h+1}') e_2}{\sigma_k^2 e_1' \left(\sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_m'\right) e_1},\end{aligned} \tag{C.5}$$

According to the above formulation, as the forecast horizon, h, extends, both β_h and β_h^k converge to zero at long horizons since Θ_{m+h+1} converges to zero when h is sufficiently large. Assuming that there are K shocks in total, we have

$$\begin{aligned}\beta_h &= \frac{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_{m+h+1}') e_2}{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_m') e_1} \\ &= \frac{\sum_{k=1}^{K} \sigma_k^2 e_1' (\sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_{m+h+1}') e_2}{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_m') e_1} \\ &= \sum_{k=1}^{K} \beta_h^{(k)} \frac{(\sigma_k^2 e_1' (\sum_{m=0}^{\infty} \Theta_m e_k e_k' \Theta_m') e_1)}{e_1' (\sum_{m=0}^{\infty} \Theta_m \Sigma_\epsilon \Theta_m') e_1} \\ &= \sum_{k=1}^{K} \beta_h^{(k)} \frac{Var(i_t^* - i_t | \epsilon_t^k)}{Var(i_t^* - i_t)},\end{aligned} \tag{C.6}$$

where the second equation can be derived from the fact that Σ_ϵ is a diagonal matrix with σ_k^2 along its diagonal.

Equations C.4 and C.5 shows that the dynamic Fama coefficients can be obtained once the moving average representation of the DSGE model is obtained. In the subsequent sections, we deduct the form of moving average coefficient for two scenarios: DSGE models characterized by full-information rational expectations, and DSGE models with distorted belief as in Candian and De Leo (2023).

C.2 DSGE Models under Full-information Rational Expectations

As shown in Fernández-Villaverde et al. (2007), DSGE models under full information rational expectation have state-space representations in the following form:

$$\begin{aligned} x_{t+1} &= Ax_t + B\epsilon_t \\ y_t &= Cx_t + D\epsilon_t. \end{aligned}$$

The state space model implies a moving average representation of the DSGE model in the form:

$$y_t = \sum_{m=0}^{\infty} CA^m B\epsilon_{t-m-1} + D\epsilon_t, \tag{C.7}$$

so that the moving average coefficients are

$$\Theta_m = \begin{cases} D, & \text{when } m = 0 \\ CA^m B, & \text{when } m > 0. \end{cases}$$

C.3 DSGE Model with Distorted Beliefs

The solution of the DSGE model in Candian and De Leo (2023) is formulated as:

$$x_{t+1} = Ax_t + B\eta_t \tag{C.8}$$

$$y_t = Cx_t + D\eta_t. \tag{C.9}$$

With the assumption of shock mis-perception and over-extrapolative belief, η_t is the expectation error that can be expressed as

$$\begin{aligned}\eta_t &= x_t - x_{t|t-1} \\ &= \Phi_1\eta_{t-1} + \Phi_2\eta_{t-2} + \mathcal{M}\varepsilon_t - \widetilde{\rho}\mathcal{M}\varepsilon_{t-1},\end{aligned} \tag{C.10}$$

where Φ_1 and Φ_2 are diagonal matrix with $(1-\kappa)\widetilde{\rho}_k + \rho_k$ and $-(1-\kappa)\widetilde{\rho}_k\rho_k$ along the diagonal respectively. $\mathcal{M}$ measures the standard deviation of true shocks, ρ_k is the true persistence of the k-th shock, $\widetilde{\rho}_k$ is the perceived shock persistence, and κ is the steady-state Kalman gain for belief updates.

In the following, we first attempt to express the expectation errors as a moving average process of the structural shocks. To do that, we rewrite equation (C.10) into the matrix form:

$$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \eta_t \\ \eta_{t-1} \end{bmatrix} = \begin{bmatrix} \Phi_1 I & \Phi_2 I \\ I & 0 \end{bmatrix}\begin{bmatrix} \eta_{t-1} \\ \eta_{t-2} \end{bmatrix} + \begin{bmatrix} \mathcal{M} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} + \begin{bmatrix} -\widetilde{\rho}\mathcal{M} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_{t-1} \\ 0 \end{bmatrix}$$

Define

$$U = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, V = \begin{bmatrix} \Phi_1 I & \Phi_2 I \\ I & 0 \end{bmatrix}, W = \begin{bmatrix} \mathcal{M} & 0 \\ 0 & 0 \end{bmatrix}, X = \begin{bmatrix} -\widetilde{\rho}\mathcal{M} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \eta_t \\ \eta_{t-1} \end{bmatrix} = U^{-1}V\begin{bmatrix} \eta_{t-1} \\ \eta_{t-2} \end{bmatrix} + U^{-1}W\begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} + U^{-1}X\begin{bmatrix} \varepsilon_{t-1} \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \eta_t \\ \eta_{t-1} \end{bmatrix} = \sum_{i=0}^{\infty}\left(U^{-1}V\right)^i U^{-1}W\begin{bmatrix} \varepsilon_{t-i} \\ 0 \end{bmatrix} + \sum_{i=0}^{\infty}\left(U^{-1}V\right)^i U^{-1}X\begin{bmatrix} \varepsilon_{t-i-1} \\ 0 \end{bmatrix}$$

$$\Rightarrow \eta_t \triangleq \sum_{i=0}^{\infty} \Psi_i \varepsilon_{t-i} \tag{C.11}$$

$$\text{where} \quad \Psi_i = \begin{cases} \begin{bmatrix} I & 0 \end{bmatrix} U^{-1} W \begin{bmatrix} I \\ 0 \end{bmatrix} & \text{when } i = 0 \\ \begin{bmatrix} I & 0 \end{bmatrix} \left[\left(U^{-1}V\right)^i U^{-1} W + \left(U^{-1}V\right)^{i-1} U^{-1} X \right] \begin{bmatrix} I & 0 \end{bmatrix} & \text{when } i > 0 \end{cases}$$

As Equations C.8 and C.9 imply that

$$y_t = \sum_{m=0}^{\infty} C A^m B \eta_{t-m-1} + D \eta_t, \tag{C.12}$$

the moving average representation of the endogenous vector is expressed as

$$y_t = C \sum_{m=0}^{\infty} A^m B \sum_{i=0}^{\infty} \Psi_i \varepsilon_{t-m-i-1} + D \sum_{m=0}^{\infty} \Psi_m \varepsilon_{t-m} \tag{C.13}$$

$$\triangleq \sum_{m=0}^{\infty} \Theta_m \varepsilon_{t-m}, \tag{C.14}$$

where

$$\Theta_m = \begin{cases} D\Psi_0, & \text{when } m = 0 \\ \sum_{k=0}^{m-1} C A^k B \Psi_{m-k-1} + D \Psi_m, & \text{when } m > 0. \end{cases}$$