Labor Supply and the Optimality of Social Security

by Shantanu Bagchi

September, 2014
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Abstract

Traditional economic theory predicts that unfunded social security can be justified on the basis of its ability to efficiently finance retirement, and also for its ability to provide insurance against mortality risk and uninsurable shocks to labor income. In this paper, I demonstrate that the quantitative importance of the traditional roles of social security depends on how household labor supply responds to social security. I build a calibrated general-equilibrium model where social security has a large welfare-improving role, and I show that the distortionary effect on households’ labor hours erases virtually all the welfare gains from social security. I also find that this result is robust within the range of labor supply elasticities usually encountered in the macroeconomic literature.

JEL Classification: E21, H55, J22

Keywords: labor supply; social security; mortality risk; productivity shock; insurance; elasticity

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*I would like to thank Kartik Athreya, Frank Caliendo, Kaiji Chen, Jim Feigenbaum, Scott Findley, Carlos Garriga, and Jorge Alonso Ortiz for useful comments and suggestions. An earlier draft of this paper was a part of my Ph.D. dissertation, and was also presented at the 2010 Midwest Macroeconomics Meetings.

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1 Introduction

Traditional economic theory predicts that unfunded social security can be justified on the basis of its ability to efficiently finance retirement, and also for its ability to provide insurance against risks that may otherwise be uninsurable. When households are rational, forced participation in social security crowds out private saving, and therefore reduces the aggregate capital stock. This negative impact of social security on aggregate capital can be useful in eliminating dynamic inefficiency from an economy. When households face uncertain lifetimes and missing annuity markets, social security can partially substitute for annuities and provide some measure of insurance against mortality risk. Finally, social security can also provide partial insurance against shocks to labor productivity by progressively linking retirement benefits to work-life income.

In this paper, I demonstrate that in an economy with rational households, the quantitative importance of the traditional roles of social security depends on how labor supply responds to social security. Social security increases the marginal tax rate on labor income, and therefore has a negative impact on the households’ labor hours over the life cycle. I show that this distortionary effect can be large enough to erase virtually all the traditional welfare gains from social security. Even in an environment where social security is highly desirable, accounting for these distortions leads to a decline of more than 80% in the optimal payroll tax rate. I also find that this result is quite robust: within the range of labor supply elasticities generally accepted in the macroeconomic literature, accounting for these distortions leads to a 75-92% decline in the optimal payroll tax rate for social security.

To examine the quantitative importance of the labor supply distortions, I build a general-equilibrium model in which social security has a large welfare-improving role. First, I assume that households in the model experience mortality risk and permanent shocks to labor productivity, but do not have access to markets where they can purchase insurance against these risks. Social security is highly beneficial in this environment, as it can partially replace both of these missing markets. Second, because markets for annuities do not exist in the model, households leave (and receive) accidental bequests in equilibrium. Because these bequests are paid out of the assets of deceased households, social security can reduce private saving and potentially crowd out these bequests (Caliendo et al., 2013). However, I assume that social security has no effect on the equilibrium value of these bequests, thereby ignoring a fundamental negative impact on household utility. Together, these two assumptions provide a model environment where social security can lead to substantial welfare improvements.

Firms in the model produce output using labor and capital, and the factor markets are perfectly competitive. The government runs two programs in the model: the social security program financed through a payroll tax, and a general tax-and-transfer program financed through a tax on labor income. I calibrate the model to reasonably match the U.S. economy, and then search for the payroll tax rate that maximizes average ex-ante expected utility. Finally, I compare the optimal tax rates under two different experiments: (i) holding household labor supply constant at the baseline level, and (ii) allowing household labor supply to adjust to the changing payroll tax rates.

I find that for the baseline calibration, social security does have a large welfare-improving role in the model economy: holding household labor supply constant, expected utility is maximized at the payroll tax rate of 12.4%, which is even larger than the current U.S. rate of 10.6%. However, allowing labor supply to respond to social security leads to a considerably lower optimal payroll tax rate: only 2.1%. Therefore, even in an environment where social security is highly beneficial,

\footnote{In their life cycle analysis of social security, Imrohoroglu et al. (1995) demonstrate that once this channel is shut-off, the optimal replacement rate increases from 30% to 40%. See Section 5 for further discussion.}
the distortions to households’ labor hours erases virtually all of its welfare gains.

Because the distortions in labor hours are largely determined by a household’s labor supply elasticity, I repeat this exercise for two other baseline calibrations of the model that feature different values for these elasticities. First, I consider a utility function that is separable in consumption and leisure, and then I also consider a calibration with elasticity values close to the lower bound of the accepted parameter space in macroeconomics. In each case, I find that accounting for the labor supply adjustments yields an optimal payroll tax rate that is significantly lower than the optimal tax rate when labor supply is held constant.

Several studies have examined the ability of the traditional roles of social security in justifying the size of the current social security program in the U.S. Abel (1985) and Hubbard and Judd (1987) find a welfare-improving role for U.S. social security in a model with mortality risk and closed annuity markets. Hubbard and Judd (1987) also find that the welfare gains are significantly reduced or even eliminated if borrowing constraints are introduced into the model. İmrohoroglu et al. (1995) examine the optimality of unfunded social security in a life-cycle economy with mortality risk, missing annuity markets, idiosyncratic employment risk, and borrowing constraints. Calibrating the model to match some key features of the U.S. economy, they find that the optimal social security arrangement features a replacement rate of 30%. However, none of these studies account for the distortionary effect of social security on households’ labor hours.

There is considerable evidence that tax-and-transfer programs, in general, have quantitatively important effects on household labor supply. Prescott (2004) demonstrated that differences in marginal tax rates alone explain most of the differences in labor supply within the advanced industrial economies (the G7 countries). Rogerson and Wallenius (2009) extend this result to an overlapping-generations environment where households choose both the fraction of lifetime spent in employment, and also the fraction of the period time endowment spent working while employed. Studies that specifically look at the effect of social security on labor supply include Ortiz (2009) and Wallenius (2009). Ortiz (2009) finds that differences in the institutional features of social security account for 90% of the differences in employment to population ratios at ages 60-64 in the OECD. Similarly, Wallenius (2009) finds that the cross-country differences in social security programs account for 35-40% of the differences in aggregate hours worked between the U.S., and Belgium, France, and Germany.

Starting with Auerbach and Kotlikoff (1987), notable studies in the dynamic pension reform literature, such as De Nardi et al. (1999), Nishiyama and Smetters (2005), Conesa and Garriga (2008), and Kitao (2011), have all used models that account for the effect of social security on labor supply. Most notably, Imrohoroglu and Kitao (2009) show that the effects of social security reform on aggregate labor supply are invariant with respect to households’ labor supply elasticity. They find that even though reforms under different values of labor supply elasticity lead to different allocations of hours over the life cycle, the aggregate effects are very similar. However, Nishiyama and Smetters (2008) appears to be the only study to have examined the welfare consequences of these distortions, albeit in a different context. Nishiyama and Smetters (2008) demonstrate that the explicitly progressive formulation of the social security benefits in the U.S. may not be optimal, as the higher replacement rates for the poor introduce various marginal tax rates that distort labor supply. Even though the progressive formulation provides partial insurance against shocks to labor income, they find that these two effects roughly cancel each other, so that the optimal replacement rate structure for U.S. social security is fairly flat. The findings of this paper are very similar: the distortionary effects on labor supply are a fundamental cost of social security, which can be large enough to erase most of the benefits that the program is traditionally known to offer.

2Other studies that arrive at a similar conclusion include Ohanian et al. (2008) and Rogerson (2008).
This paper is also related to a large literature that examines, both theoretically and quantitatively, the importance of the forced saving, insurance, and redistributive roles of unfunded social security in an environment where households operate under bounded rationality. Feldstein (1985) was the first to demonstrate that the optimal level of social security benefits in an economy with myopic households is strictly positive. Several later studies, such as Cremer et al. (2008, 2009), have studied the interaction of myopia with factors such as information asymmetry and wage heterogeneity. Time-inconsistent behavior has also been offered as a justification for social security, such as short-term planning (Findley and Caliendo, 2009), self-control preferences (Kumru and Thanopoulos, 2008), and hyperbolic discounting (İmrohoroğlu et al., 2003; Fehr et al., 2008). While it is not clear which of these explanations does the best job of justifying the current size of social security programs across the industrialized world, it seems, at least quantitatively, that studies which account for the distortionary effect of social security on labor supply, typically find a smaller welfare-improving role for social security. The findings of this paper lend further support to this observation.

The rest of the paper is organized as follows: Section 2 introduces the model and Section 3 describes the baseline calibration. In Section 5, I compare the optimal payroll tax rates for social security under the two experiments described above. I examine the sensitivity of the findings in Section 6, and Section 7 concludes.

2 The model

Consider an overlapping generations economy with life-cycle permanent-income households, where at each instant a new cohort is born and the oldest cohort dies. Cohort size grows at the rate of $n$ per annum, and fraction $f_i$ of newborns in a cohort receives a permanent productivity shock $\varphi_i$, where $\sum_i f_i = 1$. Maximum lifespan is $T$ and households face an unconditional probability $Q(s)$ of surviving to age $s$. Therefore, total population at date $t$

$$P(t) = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)$$

(1)

grows at rate $n$ over time, where $N(t-s)$ is the size of the cohort born at date $t-s$.

Over the life cycle, households accumulate a risk-free asset: physical capital. Private annuities markets are closed by assumption, because of which households are unable to fully insure themselves against mortality risk. At each date, surviving households also receive accidental bequests from the deceased households. Households earn labor income if they work, and from age $T_i$ onwards, they also receive social security benefits that are positively linked to their work-life income. Firms operate competitively and produce output using capital, labor and a constant returns to scale technology. Finally, the government runs two programs: social security financed through a payroll tax ($\tau_{ss}$), and a general tax and transfer program financed through an income tax ($\tau_y$). There is also technological progress at the rate of $g$ per annum.

Assuming closed private annuities markets is standard in this line of literature, and is also empirically consistent because in reality very few people annuitize. This phenomenon is referred to as the “non-annuitization” puzzle, because a standard life-cycle model predicts that households ought to invest exclusively in annuities if they are fairly priced. Explanations behind this puzzle include existence of pre-annuitized wealth in retirees’ portfolios, actuarially unfair prices, bequest motives, and uncertain health expenses. See, for example, studies such as Pashchenko (2010), Dushi and Webb (2004), Mitchell et al. (1999), Lockwood (2012), and Turra and Mitchell (2004).
2.1 Preferences

Period utility depends on both consumption \((c)\) and the fraction of total time endowment enjoyed in leisure \((l)\). It has the standard CIES form

\[
    u(c, l) = \begin{cases} 
        \frac{\left(\frac{c^{\eta - \sigma}}{1 - \sigma} \right)^{1 - \sigma}}{\ln \left(\frac{c^{\eta - \sigma}}{1 - \sigma} \right)} & \text{if } \sigma \neq 1 \\
        \ln \left(\frac{c^{\eta - \sigma}}{1 - \sigma} \right) & \text{if } \sigma = 1 
    \end{cases}
\]

(2)

where \(\eta\) is the share of consumption, and \(\sigma\) is the inverse of intertemporal elasticity. Expected lifetime utility from the perspective of a household of type \(i\) born at date \(t\) is

\[
    U_i = \sum_{s=0}^{T} \beta^s Q(s) u(c_i(t + s, t), l_i(t + s, t))
\]

(3)

where \(\beta\) is the discount factor. Also, since I define leisure as a fraction of the total time endowment, \(0 \leq l_i(t + s, t) \leq 1\).

2.2 Income

Conditional on survival, a household of type \(i\) born at date \(t\) earns net of taxes wage income \((1 - \tau_{ss} - \tau_y)(1 - l_i(t + s, t))w(t + s)e(s)\varphi_i\) at every age \(s\), where \(w(t + s)\) is the wage rate, and \(e(s)\) is an age-dependent efficiency endowment. Note that the permanent productivity shock \(\varphi_i\) affects the net of taxes wage income both directly and indirectly: households with a favorable productivity shock are also likely to supply more labor. After age \(T_i\), a household of type \(i\) receives social security benefits \(b_i(t + s)\) until death, and every household receives a lump-sum welfare payment \(\chi(t + s)\) every period of the life cycle.\(^4\) Each household born at date \(t\) that survives to age \(s\) also receives an accidental bequest \(B(t + s)\).

It is worth noting that in the current model, the shocks to labor productivity are ex-ante, or are realized before the agents enter the model. An alternative and widely used specification assumes that the shocks are ex-post, or are realized after the agents enter the model, and also that they persist over time.\(^5\) However, the relevant factor for the insurance role of social security is how these shocks to work-life income persist into retirement, or in other words, how strongly retirement benefits are linked to work-life income.\(^6\)

2.3 Social security and tax-and-transfer

In the model, social security provides partial insurance against mortality risk, and also against an unfavorable permanent productivity shock. The benefit at date \(t + s\) for a household with the productivity shock \(\varphi_i\) is \(b_i(t + s)\), which is a concave function of average work-life income, measured by

\[
    AWI_i = \frac{1}{T_i} \sum_{s=0}^{T_i} \left\{ 1 - l_i(t, t - s) \right\} w(t)e(s) \varphi_i
\]

(4)

\(^4\)Note that with this formulation, the current model most likely underestimates the distortionary effect of social security on labor supply. In the U.S., households can start collecting social security benefits as early as age 62, and can delay collection to as late as age 70. Based on how early or late the actual collection date is from the full-retirement age, households receive an adjustment in their benefits. These adjustments, if actuarially unfair, are an additional source of distortion to labor supply, which the current model ignores.

\(^5\)See, for example, studies such as İmrohoroglu et al. (1995), Huggett (1996), and İmrohoroglu and Kitao (2009).

\(^6\)Zhao (2011) uses a calibrated general-equilibrium model with ex-ante labor productivity shocks to examine the role of social security in explaining the rise in health-care costs in the U.S.
Note that $T_i=1$ is the retirement age of a household of type $i$, or the age at which labor supply drops to zero. Also, note that similar to the SSA’s calculations, past wages are indexed to date $t$ in computing the AWI. The general tax-and-transfer program collects a constant fraction of household earnings (which also depends on the permanent productivity shock) and pays out a uniform benefit $\chi(t+s)$ to each surviving household.\(^7\) The government balances the budget for both of these programs.

### 2.4 A household’s optimization problem

A household of type $i$ born at date $t$ faces the following optimization problem

$$\max_{c_i,l_i} \sum_{s=0}^{T} \beta^s Q(s) u(c_i(t+s),l_i(t+s))$$

subject to

$$c_i(t+s,t) + k_i(t+s+1,t) = (1+r)k_i(t+s,t) + y_i(t+s,t) + B(t+s) + \chi(t+s)$$

$$y_i(t+s,t) = (1-\tau_{ss}-\tau_y)(1-l_i(t+s,t))w(t+s)e(s)\varphi_i + \Theta(s-T_i)b_i(t+s)$$

$$0 \leq l_i(t+s,t) \leq 1$$

$$k_i(t,t) = k_i(t+T+1,t) = 0$$

where

$$\Theta(x) = \begin{cases} 
0 & x \leq 0 \\
1 & x > 0 
\end{cases}$$

is a step function.

### 2.5 Technology and factor prices

Output is produced using a Cobb-Douglas production function with inputs capital, labor and a stock of technology $A(t)$

$$Y(t) = K(t)^{\alpha} (A(t)L(t))^{1-\alpha}$$

where $A(t) = A(0)(1+g)^t$, $\alpha$ is the share of capital in total income and $A(0)$ is the initial stock of technology. Firms face perfectly competitive factor markets, which implies

$$r = MP_K - \delta = \alpha \left[ \frac{K(t)}{A(t)L(t)} \right]^{\alpha-1} - \delta$$

$$w(t) = MP_L = A(t)(1-\alpha) \left[ \frac{K(t)}{A(t)L(t)} \right]^{\alpha}$$

where $\delta$ is the depreciation rate of physical capital, and $w(t)$ is the wage rate at time $t$. In the steady-state, the wage rate grows at rate $g$ per annum, and the rate of return $r$ is constant.

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\(^7\)This is an example of a Beveridgean tax-and-transfer program, where everyone receives identical benefits.
2.6 Aggregation

Aggregate capital stock and labor supply are given by

\[ K(t) = \sum_i f_i \sum_{s=0}^T N(t-s)Q(s)k_i(t, t-s-1) \]  \hspace{1cm} (13)

\[ L(t) = \sum_i f_i \sum_{s=0}^T N(t-s)Q(s) \{1 - l_i(t, t-s)\} e(s)\varphi_i, \]  \hspace{1cm} (14)

and the budget-balancing conditions for social security and the tax-and-transfer programs are respectively

\[ \sum_i f_i \sum_{s=0}^T N(t-s)Q(s)\tau_y(1 - l_i(t, t-s)) w(t)e(s)\varphi_i = \sum_i f_i \sum_{s=0}^T N(t-s)Q(s)\chi(t). \]  \hspace{1cm} (15)

Finally, the total value of assets held by households who die on date \( t \) is given by

\[ D(t) = (1 + r) \left[ \sum_i f_i \sum_{s=0}^T \{N(t-s)Q(s) - N(t-s-1)Q(s+1)\} k_i(t, t-s-1) \right] \]

\[- \sum_i f_i \sum_{s=0}^T (N(t-s+1) - N(t-s)) Q(s)k_i(t+1, t-s). \]  \hspace{1cm} (17)

2.7 Competitive equilibrium

I characterize a competitive equilibrium in the current model by a collection of

1. cross-sectional consumption allocations \( \{c_i(t, t-s)\}_{s=0}^T \), asset allocations \( \{k_i(t, t-s)\}_{s=0}^T \),
   and labor supply allocations \( \{1 - l_i(t, t-s)\}_{s=0}^T \),
2. aggregate capital stock \( K(t) \) and labor \( A(t)L(t) \),
3. rate of return \( r \) and wage rate \( w(t) \),
4. accidental bequest \( B(t) \), and
5. social security benefits \( b_i(t) \) and welfare payments \( \chi(t) \)

that

1. solves the households’ optimization problems,
2. equilibrates the factor markets,
3. ensures that the total value of accidental bequests \( B(t)P(t) \), is equal to the total value of assets held by the deceased households \( D(t) \),
4. balances the social security and the tax-and-transfer program budgets, and
which is equal to the total income earned from capital and labor at time \( t \). In equilibrium, total expenditure at time \( t \) equals consumption plus investment (net of depreciation), which is equal to the total income earned from capital and labor at time \( t \).

\[
C(t) + K(t + 1) - (1 - \delta)K(t) = C(t) + (n + g + ng + \delta)K(t) = \text{w}(t)L(t) + (r + \delta)K(t) = Y(t)
\]

Also, since I focus only on steady-state analysis, I set \( t = 0 \) and normalize initial newborn cohort size and technology to \( N(0) = A(0) = 1 \).

### 3 Calibration

To ensure that the baseline equilibrium matches some key aspects of U.S. macroeconomic data, I use empirical evidence from various sources to assign values to the model’s parameters. A population growth rate of \( n = 1\% \) is consistent with the U.S. demographic history, and I set the rate of technological progress to \( g = 1.56\% \), which is the trend growth rate of per-capita income in the postwar U.S. economy (Bullard and Feigenbaum, 2007). I assume that households enter the model at actual age 25, which corresponds to the model age of zero. I obtain the survival probabilities from Feigenbaum’s (2008) sextic fit to the mortality data in Arias (2004), which is given by

\[
\ln Q(s) = -0.01943039 + (-3.055 \times 10^{-4}) s + (5.998 \times 10^{-6}) s^2 \\
+ (-3.279 \times 10^{-6}) s^3 + (-3.055 \times 10^{-8}) s^4 + (3.188 \times 10^{-9}) s^5 \\
+ (-5.199 \times 10^{-11}) s^6
\]

where \( s \) is model age. The 2001 U.S. Life Tables in Arias (2004) are reported up to actual age 100, so I set the maximum model age to \( \bar{T} = 75 \). The resulting survivor function is plotted in Figure 1. Under these survival probabilities, the model life expectancy at birth turns out to be about 79 years.

Also, I set the model benefit eligibility age to \( T_b = 41 \), which corresponds to the current actual full retirement eligibility age of 66 in the U.S. Following Imrohoroglu et al. (1995) and Conesa and Garriga (2008), I parameterize the efficiency endowment profile \( e(s) \) using data from Hansen (1993). However, it is well known that efficiency measured from wage data suffers from sample selection bias, especially at the later ages when a large number of households begin to retire. For this reason, I fit a quartic polynomial to the efficiency data in Hansen (1993) only for ages 25-65, which gives

\[
\ln e(s) = -3.273 \times 10^{-5} + (3.7484 \times 10^{-2}) s + (-1.7541 \times 10^{-3}) s^2 \\
+ (3.4625 \times 10^{-5}) s^3 + (-2.7949 \times 10^{-7}) s^4
\]

where \( s \) is model age and \( s \leq 40 \). Beyond actual age 65 (i.e. for \( s > 40 \)), I use the following quadratic function

\[
\ln e(s) = -f_0 - f_1 s - 0.01 s^2
\]

and parameterize \( f_0 \) and \( f_1 \) such that \( e(s) \) is continuous and once differentiable at age \( s = 40 \).\(^8\) Note that the coefficient of 0.01 on the squared term in (21) ensures that households do not continue to work beyond age 70.\(^9\) The resulting efficiency endowment profile is plotted in Figure 2.

\(^8\)The values that satisfy these conditions are \( f_0 = 15.4789 \) and \( f_1 = -0.7918 \).

\(^9\)Historically, the employment-to-population ratios for ages 70 and above in the U.S. have been less than 10%.
Figure 1: Survival probabilities from Feigenbaum’s (2008) sextic fit to the mortality data in Arias (2004).

Figure 2: Efficiency endowment profile fitted to data from Hansen (1993).
Social welfare payments in the U.S. are made in various forms. Payments from the OASI, Medicare and the Supplemental Security Income (SSI) programs are conditional on retirement, whereas payments from programs such as Food Stamps, hospital and medical care (excluding Medicare) and housing are not. The OASI benefit annuity in the U.S. is a concave (piecewise linear) function of work-life income. The Social Security Administration measures what is known as the Average Indexed Monthly Earnings (AIME) for every covered individual, and then replaces a fraction of the AIME. Depending on how large or small the AIME for an individual is relative to the average wage in the economy, the fraction that is replaced gets adjusted. For example, in 2001 the OASI benefit annuity in the U.S. was 90% of the AIME for the first $561, 32% of the next $3381, and 15% of the remaining up to the maximum creditable earnings. As shown by Huggett and Ventura (1999), these dollar amounts come out to be 20%, 124% and 247% of the average wage in the economy. These percentage amounts are referred to as the “bend points” of the benefit rule, and I take them directly to the model. Note that the progressivity in the benefit rule is captured by the fact that the replacement rate is decreasing in the AIME (see Figure 3).

To calibrate the labor income tax rate \(\tau_y\), I use data on social welfare expenditures in the U.S.\(^\text{10}\) According to the SSA, total social welfare spending in the U.S. is roughly 21% of GDP. Deducting OASI, Disability, Medicare, Railroad Retirement benefits, and Public Employee Retirement benefits, the remaining social welfare spending turns out to be 12.28% of GDP. This includes items such as food stamps, hospital and medical care (excluding Medicare), veterans programs, education, public housing, and several other social welfare services and benefits. I calibrate the income tax rate such that total payments from the tax-and-transfer program in the model at date \(t\), measured as \(\sum_i J_i \sum_{s=0}^{T} N(t-s)Q(s)\chi(t)\), matches 12.28% of GDP.

\(^{10}\)See http://www.socialsecurity.gov/policy/docs/progdesc/sspusr/appeni.pdf.
The historically observed value of capital’s share in total income in U.S. ranges between 30-40%, so I set $\alpha = 0.35$. To calibrate the permanent productivity shock and its distribution, I follow Zhao (2011) and assume that $\ln \varphi \sim N(0, \sigma^2_{\varphi})$, and then set $\sigma^2_{\varphi} = 0.65$ to be consistent with the empirical estimates of Heathcote et al. (2010). Also, I use Gaussian quadrature to transform the continuous distribution into a 5-point discrete distribution for computational convenience. Finally, I set the payroll tax rate for social security in the baseline calibration to $\tau_{ss} = 0.106$.

Once all the observable parameters have been assigned empirically reasonable values, I calibrate the unobservable preference parameters $\sigma$ (IEIS), $\beta$ (discount factor) and $\eta$ (share of consumption in period utility), and the depreciation rate $\delta$ such that the baseline equilibrium jointly matches the following targets:

- an equilibrium capital-output ratio of 3.0,
- an average of 34 hours per week spent on market work between ages 25-55, and
- a ratio of aggregate consumption expenditure to income of 70%.

The unobservable parameter values under which the baseline equilibrium reasonably matches the above targets are reported in Table 1. Note that with leisure in period utility, the relevant inverse elasticity for consumption is $\sigma^c = 1 + \eta(\sigma - 1) = 1.76$, which lies within the range frequently encountered in the literature. The model-generated values for the targets under the baseline calibration are reported in Table 2, and the cross-sectional means of consumption and labor hours are reported in Figures 4 and 5.  

### Table 1: Unobservable parameter values under the baseline calibration.

<table>
<thead>
<tr>
<th></th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.0211</td>
<td>0.803</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.2523</td>
<td>0.803</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0744</td>
<td>0.803</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.189</td>
<td>0.803</td>
</tr>
</tbody>
</table>

### Table 2: Model performance under the baseline calibration.

<table>
<thead>
<tr>
<th></th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital-output ratio</td>
<td>3.00</td>
<td>3.01</td>
</tr>
<tr>
<td>Consumption expenditure to income ratio</td>
<td>0.70</td>
<td>0.699</td>
</tr>
<tr>
<td>Avg. hours of market work per week between ages 25-55</td>
<td>34</td>
<td>33.8</td>
</tr>
<tr>
<td>${\sum_i f_i \sum_{s=0}^{T} N(t - s)Q(s)\chi(t)} / Y(t)$</td>
<td>0.1228</td>
<td>0.1229</td>
</tr>
</tbody>
</table>

### 4 Welfare measure

Once the model is calibrated, the next step is to use this model to identify the optimal payroll tax rate for social security. To do this, I define the following measure of welfare:

$$W = \sum_i f_i \sum_{s=0}^{T} \beta^s Q(s)u(c_i(t + s, t), l_i(t + s, t)).$$  \hspace{1cm} (22)

From the perspective of a household of type $i$ born at date $t$, $\sum_{s=0}^{T} \beta^s Q(s)u(c_i(t + s, t), l_i(t + s, t))$ captures the ex-ante expected lifetime utility, so $W$ simply gives the weighted average of all such $\{\sum_i f_i \sum_{s=0}^{T} N(t - s)Q(s)\chi(t)\} / Y(t)$.

\[\text{The cross-sectional mean of a variable } x_i(t, t - s) \text{ is calculated using the formula } \bar{x}(t, t - s) = \sum_i f_i x_i(t, t - s).\]
Figure 4: Cross-sectional mean of consumption (normalized by consumption at age 25) under the baseline calibration.

Figure 5: Cross-sectional mean of labor hours per week under the baseline calibration.
expected utilities of the different household types in the economy. As discussed earlier, there are three ways in which social security can improve welfare in this economy. First, social security can crowd out private saving, reduce capital accumulation, increase the equilibrium rate of return, and therefore push it towards the golden rule level of capital stock. Second, because annuity markets are closed, social security can provide partial insurance against mortality risk. Finally, the permanent productivity shock affects household labor income, both directly and indirectly. Holding labor hours constant, a household with an unfavorable productivity shock earns less than a household at the same age with a good productivity shock. Additionally, a household with an unfavorable productivity shock also works fewer hours, as it faces a lower opportunity cost of leisure. The social security benefit-earning rule can provide partial insurance against such a shock.

5 The optimal payroll tax rate

Notice that the competitive equilibrium definition in Section 2.7 includes the accidental bequest $B(t)$ as an equilibrium object, and the restriction $B(t)P(t) = D(t)$ as an equilibrium condition. This captures the fact that the accidental bequests to the surviving households are paid out of the assets of the deceased. However, because social security crowds out private saving, through this restriction it has a negative impact on the equilibrium accidental bequest, and therefore on household utility.\(^{12}\)

To ensure that the model environment allows social security to have a large welfare-improving role, I shut off this channel by assuming that the accidental bequest is inelastic with respect to the payroll tax rate for social security. Theoretically, this amounts to excluding the equilibrium accidental bequest condition $B(t)P(t) = D(t)$ from the computations while searching for the optimal payroll tax rate.\(^{13}\) For this reason, the total value of accidental bequests, $B(t)P(t)$, diverges from the total value of assets held by the deceased households, $D(t)$, for all tax rates but $\tau_{ss} = 0$.\(^{106}\)

Then, I compute the optimal payroll tax rate under two experiments:

- **Case 1:** holding the households’ labor hours fixed at the baseline level, and
- **Case 2:** allowing the labor hours to respond to the changing tax rates.

Table 3 shows how the welfare measure changes as I compute new equilibria of the model with values for the payroll tax rate starting from $\tau_{ss} = 0$ under Case 1 (the second column is the average replacement rate). The value at which welfare is maximized is $\tau_{ss} = 0.124$, which is even larger than the current U.S. rate of 10.6%. Notice that without social security (i.e. with $\tau_{ss} = 0$), the economy is dynamically inefficient under Case 1: the equilibrium rate of return on physical capital is $r = 1.58\%$, which is lower than the rate of growth of the economy $(1 + n)(1 + g) - 1 = 2.58\%$. Payroll tax rates between 3% and 4% eliminate the dynamic inefficiency, but the optimal rate is considerably larger. This is because social security has two additional roles in the current model: providing partial insurance against mortality risk, and also against unfavorable shocks to productivity. Also, note that because the households’ labor hours are held fixed at the baseline under Case 1, the average hours per week are invariant to the tax rate.

\(^{12}\)Caliendo et al. (2013) actually show that in a partial equilibrium endowment economy, this accidental bequest channel completely offsets any positive effect that social security has on household utility.

\(^{13}\)Imrohoroglu et al. (1995) shut off this accidental bequest channel using a slightly different strategy: they assume that the bequests are destroyed and provide no utility to the surviving households.

\(^{14}\)Note that with this modification, total expenditure in the model $C(t) + (n + g + ng + \delta)K(t) + D(t)$ is still equal to total income $w(t)\bar{L}(t) + (r + \delta)K(t) + B(t)P(t)$, but not equal to total output for any tax rate but $\tau_{ss} = 0.106$. 
Table 3: The optimal payroll tax rate for social security under Case 1.

<table>
<thead>
<tr>
<th>$\tau_{ss}$</th>
<th>Avg. rep. rate</th>
<th>$r$</th>
<th>$K/Y$</th>
<th>$C/Y$</th>
<th>Avg. hrs. per week</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.00</td>
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<td>3.88</td>
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<td>2.93</td>
<td>0.71</td>
<td>33.8</td>
<td>-89.405</td>
</tr>
<tr>
<td>0.124</td>
<td>0.68</td>
<td>0.045</td>
<td>2.92</td>
<td>0.71</td>
<td>33.8</td>
<td>-89.404</td>
</tr>
<tr>
<td>0.126</td>
<td>0.69</td>
<td>0.046</td>
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<td>0.71</td>
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Table 4: The optimal payroll tax rate for social security under Case 2.

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<th>$C/Y$</th>
<th>Avg. hrs. per week</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
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<td>0.000</td>
<td>0.00</td>
<td>0.025</td>
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<td>0.61</td>
<td>36.74</td>
<td>-87.806</td>
</tr>
<tr>
<td>0.010</td>
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<td>0.027</td>
<td>3.46</td>
<td>0.63</td>
<td>36.47</td>
<td>-87.700</td>
</tr>
<tr>
<td>0.015</td>
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<td>0.028</td>
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<td>0.63</td>
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<td>-87.674</td>
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<tr>
<td>0.019</td>
<td>0.11</td>
<td>0.028</td>
<td>3.40</td>
<td>0.63</td>
<td>36.22</td>
<td>-87.657</td>
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<tr>
<td>0.021</td>
<td>0.12</td>
<td>0.029</td>
<td>3.39</td>
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<td>36.17</td>
<td>-87.655</td>
</tr>
<tr>
<td>0.023</td>
<td>0.14</td>
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<td>3.38</td>
<td>0.64</td>
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<td>-87.656</td>
</tr>
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<td>0.025</td>
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</tr>
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<td>0.64</td>
<td>35.93</td>
<td>-87.665</td>
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<td>-87.697</td>
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<td>3.24</td>
<td>0.66</td>
<td>35.38</td>
<td>-87.810</td>
</tr>
</tbody>
</table>

Figure 6 shows the effect of social security on cross-sectional consumption under Case 1, i.e. when labor supply is held fixed at the baseline. It is clear from the figure that higher payroll tax rates lead to steeper consumption profiles. The evolution of consumption over the life cycle is given by

$$\frac{c(t+s+1,t)}{c(t+s,t)} = \left(\frac{(1+g)e(s+1)}{e(s)}\right)^{-\frac{(1-\eta)(1-\sigma)}{\sigma}} \left(\frac{Q(s+1)}{Q(s)}\beta(1+r)\right)^{1/\sigma},$$

which shows that consumption changes more rapidly between two successive ages if the rate of return ($r$) is higher. A higher payroll tax rate reduces private saving and increases the equilibrium rate of return, because of which consumption profiles are much steeper. Note that because the hours are held fixed at the baseline, consumption under Case 1 drops due to retirement at the same ages as the baseline (see Figure 6).

Next, I compute the optimal payroll tax rate under Case 2, i.e. I allow household labor supply to respond to the changing tax rates. Table 4 shows how the welfare measure changes as I compute new equilibria of the model with values for the payroll tax rate starting from $\tau_{ss} = 0$. The value at which welfare is maximized is $\tau_{ss} = 0.021$, which is significantly smaller than 12.4% under Case 1.

Even under Case 2, the economy is dynamically inefficient in the absence of social security: the rate of return with $\tau_{ss} = 0$ is 2.47%. However, in this case tax rates between 0% and 1% eliminate the dynamic inefficiency: the equilibrium rate of return at the optimum is $r = 2.88\%$. The negative impact of social security on household labor supply can be clearly seen in the sixth column of Table 4: higher tax rates lead to a steady decline in the average hours worked per week. On the aggregate, labor supply under Case 2 declines by about 4% between $\tau_{ss} = 0$ and $\tau_{ss} = 0.021$, and
Figure 6: Cross-sectional means of consumption with $\tau_{ss} = 0$, 0.124, and 0.106 under Case 1.

by almost 18% between $\tau_{ss} = 0$ and $\tau_{ss} = 0.124$.

Figures 7 and 8 show the effect of social security on cross-sectional consumption and hours per week under Case 2. Consumption profiles are steeper for higher tax rates even under Case 2, but the declines in work-life consumption are significantly larger. This is because with higher payroll tax rates, hours per week after age 40 are considerably lower. Because the endowment profile peaks at about age 50 and declines at a relatively slow rate thereafter, the loss of hours during the most productive ages overpowers the slight gains prior to age 40 (see Figure 8). The welfare consequences of these distortions to labor supply are large, because social security brings about a decline in hours during the ages at which the households are most productive.

To summarize, the baseline model predicts that the distortionary effect of social security on household labor supply erases a large part of the welfare gains from the program. Accounting for this distortionary effect, the optimal payroll tax rate is only 2.1%, which is less than a fifth of the optimal tax rate of 12.4% when the distortions are not accounted for.

6 Sensitivity analysis

The distortionary effect of social security on labor depends on the value of households' labor supply elasticity. With the given utility function, the Frisch elasticity of labor supply at age $s$ for a household of type $i$ born at date $t$ is given by

$$
\varepsilon_{i}(t+s,t) = \frac{l_{i}(t+s,t)}{1 - l_{i}(t+s,t)} \frac{1 + \eta(\sigma - 1)}{\sigma}.
$$

(24)
Figure 7: Cross-sectional means of consumption with $\tau_{ss} = 0$, 0.021, and 0.106 under Case 2.

Figure 8: Cross-sectional means of hours per week with $\tau_{ss} = 0$, 0.021, and 0.106 under Case 2.
I report the cross-sectional mean Frisch elasticity by age in the baseline calibration in Figure 9.\footnote{I report the Frisch elasticity only up to age 54 because as households begin to retire, the value of the elasticity goes to infinity.} It is clear from the figure that the elasticities are significantly larger than zero: the lowest value is about 1.5 occurring at roughly age 33, and the average is 2.3 between ages 25-54. However, as

\[ \sigma = 4 \]

**Figure 9:** Cross-sectional mean of Frisch elasticity of labor supply under the baseline calibration.

...equation (24) demonstrates, the elasticity of labor supply depends on the values of the preference parameters used to calibrate the model. More importantly, in the baseline calibration of the model, the IEIS and the discount factor are not separately identified, which implies that there are other values of these parameters for which the model matches the targets just as well.\footnote{The identification problem arises because there are three unknown preference parameters in the baseline model, but only two targets to calibrate them. One way to uniquely identify the three parameters is to include another targets, such as life-cycle consumption data, in the calibration. However, the inability of models of this type in matching consumption data is well known. See Bullard and Feigenbaum (2007) and Bagchi and Feigenbaum (2013) for further details.} How sensitive is the distortionary effect of social security on labor to the values of households’ labor supply elasticity?

To examine this, I explore two alternative calibrations of the model: one with $\sigma = 1$ or log utility, and another with $\sigma = 6$. These values of the IEIS roughly bracket-off the range that is typically used in the macro-calibration literature, and also yield a fairly wide range of Frisch elasticities. In Table 5, I report the values of the unobservable parameters for which the model reasonably matches the targets in each case, and in Table 6, I report the model performance under those parameter values. With log utility, the marginal utilities of consumption and leisure are independent of each other, because of which labor supply is considerably more elastic than with $\sigma = 4$. The cross-sectional mean Frisch elasticity with $\sigma = 1$ is at its minimum value of about 3...
Table 5: Unobservable parameter values with $\sigma = 1$ and $\sigma = 6$.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\beta$</th>
<th>$\eta$</th>
<th>$\delta$</th>
<th>$\tau_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9899</td>
<td>0.2461</td>
<td>0.0744</td>
<td>0.189</td>
</tr>
<tr>
<td>6</td>
<td>1.0420</td>
<td>0.2592</td>
<td>0.0744</td>
<td>0.189</td>
</tr>
</tbody>
</table>

Table 6: Model performance with $\sigma = 1$ and $\sigma = 6$.

\[
\left\{ \sum_i f_i \sum_{s=0}^T N(t-s)Q(s)\chi(t) \right\}/Y(t) = 0.1228 \\
0.1229 \\
0.1229 \\
0.1228
\]

Table 7: The optimal payroll tax rate for social security under Case 1 with $\sigma = 1$.

For each of these two alternative calibrations, I first search for the optimal payroll tax rate for social security, holding the household labor hours fixed at the respective baselines (Case 1), and then repeat the exercise while allowing the labor hours to respond to the changing payroll tax rates (Case 2). Tables 7 and 8 show how the welfare measure changes with the payroll tax rates under Case 1 for $\sigma = 1$ and $\sigma = 6$ respectively. Even with log utility, the economy is dynamically inefficient without social security under Case 1, and the optimal payroll tax rate is 12.3%. Note that this is almost identical to the optimal tax rate of 12.4% under Case 1 with $\sigma = 4$. The cross-sectional means of consumption under Case 1 with log utility are plotted in Figure 10 for payroll tax rates $\tau_{ss}=0$, 0.123, and 0.106 respectively. Notice that similar to $\sigma = 4$, consumption profiles are steeper for higher tax rates, simply because social security crowds out private saving and increases the equilibrium rate of return to physical capital. However, because utility is now separable in consumption and leisure, consumption is smooth across retirement unlike $\sigma = 4$.

The effect of social security on the welfare measure under Case 1 with $\sigma = 6$ is reported in Table 8. The optimal payroll tax rate in this case is 13.3%, which is slightly higher than those under $\sigma = 1$ and $\sigma = 4$. The cross-sectional means of consumption for payroll tax rates $\tau_{ss}=0$, 0.123, and 0.106 respectively.
Figure 10: Cross-sectional means of consumption with $\sigma = 1$ and $\tau_{ss} = 0, 0.123, \text{ and } 0.106$ under Case 1.

<table>
<thead>
<tr>
<th>$\tau_{ss}$</th>
<th>Avg. rep. rate</th>
<th>$r$</th>
<th>$K/Y$</th>
<th>$C/Y$</th>
<th>Avg. hrs. per week</th>
<th>$W$</th>
</tr>
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<td>0.00</td>
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<td>34.3</td>
<td>-203.868</td>
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<td>-200.185</td>
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<td>0.130</td>
<td>0.71</td>
<td>0.047</td>
<td>2.88</td>
<td>0.72</td>
<td>34.3</td>
<td>-199.648</td>
</tr>
<tr>
<td>0.131</td>
<td>0.72</td>
<td>0.047</td>
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<td>0.72</td>
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<td>-199.646</td>
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<td>0.72</td>
<td>34.3</td>
<td>-199.647</td>
</tr>
<tr>
<td>0.140</td>
<td>0.77</td>
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<td>2.83</td>
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<td>-199.670</td>
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<td>2.78</td>
<td>0.73</td>
<td>34.3</td>
<td>-199.782</td>
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</table>

Table 8: The optimal payroll tax rate for social security under Case 1 with $\sigma = 6$.

0.133, and 0.106 under Case 1 with $\sigma = 6$ are reported in Figure 11. Because labor is less elastic with $\sigma = 6$, the drop in consumption at retirement is somewhat larger.

The next step is to compute the optimal payroll tax rate, while allowing the labor hours to respond to social security (Case 2). Tables 9 and 10 demonstrate how the welfare measure changes with the payroll tax rate under Case 2 for $\sigma = 1$ and $\sigma = 6$ respectively. With log utility, the payroll tax rate that maximizes welfare under Case 2 is 3.1%, which is significantly smaller than the corresponding optimal payroll tax rate of 12.3% under Case 1. The negative impact of social security on household labor supply can be seen in Table 9: there is a steady decline in the average hours per week as the payroll tax rate increases. Between $\tau_{ss} = 0$ and 3.1%, aggregate labor supply falls by 3%, and almost by additional 9 percentage points between $\tau_{ss} = 3.1\%$ and 12.3%. I report in Figure 12 the cross-sectional hours per week for the tax rates $\tau_{ss} = 0, 0.031, \text{ and } 0.106$ with log utility under Case 2. As before, labor supply increases at a much faster rate at early ages, but
Figure 11: Cross-sectional means of consumption with $\sigma = 6$ and $\tau_{ss} = 0$, 0.133, and 0.106 under Case 1.

Table 9: The optimal payroll tax rate for social security under Case 2 with $\sigma = 1$.  

<table>
<thead>
<tr>
<th>$\tau_{ss}$</th>
<th>Avg. rep. rate</th>
<th>$r$</th>
<th>$K/Y$</th>
<th>$C/Y$</th>
<th>Avg. hrs. per week</th>
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<td>0.66</td>
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</tr>
<tr>
<td>0.031</td>
<td>0.18</td>
<td>0.037</td>
<td>3.14</td>
<td>0.66</td>
<td>35.91</td>
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<td>0.033</td>
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<td>0.66</td>
<td>35.89</td>
<td>-13.967</td>
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<td>3.10</td>
<td>0.67</td>
<td>35.61</td>
<td>-13.977</td>
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</tbody>
</table>
higher payroll taxes lead to larger reductions in weekly hours during the peak productivity years. The cost of these distortions is large enough to reduce the optimal payroll tax rate from 12.3% under Case 1 to 3.1% under Case 2.

Table 10: The optimal payroll tax rate for social security under Case 2 with $\sigma = 6$.

<table>
<thead>
<tr>
<th>$\tau_{ss}$</th>
<th>Avg. rep. rate</th>
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<th>$C/Y$</th>
<th>Avg. hrs. per week</th>
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<td>0.00</td>
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<td>36.98</td>
<td>-191.271</td>
</tr>
<tr>
<td>0.040</td>
<td>0.24</td>
<td>0.029</td>
<td>3.38</td>
<td>0.64</td>
<td>36.60</td>
<td>-191.720</td>
</tr>
<tr>
<td>0.050</td>
<td>0.30</td>
<td>0.031</td>
<td>3.32</td>
<td>0.65</td>
<td>36.23</td>
<td>-192.305</td>
</tr>
</tbody>
</table>

With $\sigma = 6$, the optimal payroll tax rate under Case 2 is only 1.1%, which is lower than the corresponding optimal tax rates with both $\sigma = 1$ and $\sigma = 4$. Higher payroll tax rates lead to a reduction in the weekly hours even in this case, as seen in the sixth column of Table 10) and Figure 13. Aggregate labor supply in this case declines by about 2% between $\tau_{ss} = 0$ and 1.1%, and by another 15 percentage points between $\tau_{ss} = 1.1$% and 13.3%. Qualitatively, the effect of social security on labor supply with $\sigma = 6$ is also very similar to that with $\sigma = 4$ (see Figure 8).

To summarize, I find that the distortionary effect of social security on households' hours per week is not sensitive to the value of labor supply elasticity. Across the range of labor supply
elasticities usually encountered in the macroeconomic literature, accounting for these distortions leads to a 75-92% decline in the optimal payroll tax rate.

7 Conclusions

A social security program can be theoretically justified on the basis of its ability to efficiently finance retirement, and also for its ability to provide insurance against mortality risk and uninsurable shocks to labor income. In this paper, I show that the quantitative importance of these roles depends on how household labor supply responds to social security. I find that the distortionary effect of social security on households’ hours per week can be quantitatively large enough to erase virtually all the traditional welfare gains from social security. Moreover, this finding is not sensitive to the value of households’ labor supply elasticity, and it continues to hold across the range of elasticities used in the macroeconomic literature.

The importance of accounting for the effects of tax-and-transfer programs on households’ labor hours is well known in the macroeconomic literature. The findings in the current paper suggest that the same effects may be important in determining how large or small a social security program should ideally be. Social security improves welfare by eliminating over-saving and partially replacing missing insurance markets, but also distorts households’ choice of labor hours. Therefore, public policy-making on the optimality of social security should carefully account for both of these effects.
References


