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A Theory of Liberal Churches

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Abstract

There is a counterintuitive gap in the club theory of religion. While it elegantly accounts for the notable success of strict sectarian religious groups in recruiting members and maintaining commitment, it exhibits less satisfactory properties when used to account for groups requiring neither extreme nor zero sacrifice. Such corner solutions, compared to the moderate middle, are rarely observed empirically. Within the original representative agent model, moderate groups are everywhere and always a suboptimal choice for rational, utility maximizing agents. In this paper, we extend the original model to operate within a multi-agent computational context, with heterogeneous agents occupying coordinates in a two dimensional lattice, making repeated decisions over time. Our model offers the possibility of successful moderate groups, including outcomes wherein the population is dominated by moderate groups. The viability of moderate groups is a result of heterogeneous agent wages. Lower wage agents offer greater time contributions, but lesser financial contributions to groups. Higher sacrifice rates incentive greater contributions from members, but reduce private productivity and screen out other potential members with greater financial resources. Moderate groups succeed by offering an optimal balance of these countervailing forces.

Keywords. Club Theory of Religion, Liberal, Moderate, Multi-Agent Computational Model, Sacrifice, Heterogeneous Agents

JEL Codes: C63, Z12, D71

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Said one –"Folk of a surly Tapster tell
And daub his Visage with the Smoke of Hell;
"They talk of some strict Testing of us – Pish!
"He's a Good Fellow, and't will all be well."
– Rubaiyat of Omar Khayyam, translated by Edward Fitzgerald

Introduction

Moderate religious groups, at first blush, are easier to understand than are their extreme counterparts. While the peculiar behavior of members volunteering to join groups that require unproductive costs might seem to stretch the bounds of rationality, certainly membership in relatively low costs groups makes more sense. Simple economic logic informs us that small costs are preferred to large costs. Secularization hypotheses predicted the eventual demise of religion (Swatos and Christiano 1999), and under such theories, moderate religious groups, tolerant and lenient, made for a perfect transitional stage from the irrational, costly past to the secular, liberated future. This explanation has lost some of its footing amidst the persistence of religion as a robust social institution. Iannaccone's (1992) theory of utility enhancing sacrifice requirements, where sacrifice can be broadly defined as forgone extra-group (secular) opportunities, reconciled much of the dilemma regarding the attractiveness of high cost groups. Unproductive costs, he showed, can serve to screen out potential free riders and incentivize those who do join to dedicate a greater share of their productive resources to the group. This impressive piece of rational choice theorizing reconciles the behavior of the most devout, those willingly sacrificing so much of their potential productive capacity. Counter intuitively, perhaps even ironically, in its successful modeling of extreme sacrifice, the model is considerably less adept at explaining the empirical reality of successful moderate religious groups. It is the members of low cost groups that fail to conform to the rational predictions of the club theory of religion. While the devout and the devoutly secular emerge as viable outcomes, the scores of moderately dedicated, the Unitarians, ecumenical, “mainstream” Protestants and twice-a-year Catholics, to name a few, would appear mired in the suboptimal.

Liebman (1983) anticipated this conundrum of the moderate a decade earlier, attempting to posit a sociological theory of religion as naturally extremist. He took the view that religion, operating in a simplified social vacuum, would be naturally expansive, seeking to overtake the social sphere that encompassed believers, and prohibit all that could not be incorporated within it. This was to him, the easy part. It was moderate religion that he felt was left begging an explanation. Why would religion emerge as an institution that sought to limit itself? The conclusions of the original Iannaccone model of sacrifice and stigma begs a similar question. It would seem that those who prefer any amount of religious sacrifice would in fact prefer the group demand complete prohibition of all things secular, perhaps even all things outside the bounds of the congregation, regardless of religious content. In this paper I present a theoretical extension of the original Iannaccone sacrifice and stigma model that generates a distribution of religious groups across a range of potential sacrifice requirements, reconciling the model not just with the feasibility of moderate religion, but with the possibility of a variety of disparate groups coexisting within a population. The characteristics that allow for the emergent viability and dominance of moderate religious groups are congruent with the observable characteristics of populations where moderate religious groups flourish.

Religious Groups and Free Riding

While secularization theories come in a variety of forms (Swatos and Christiano 1999), economic theory would be more inclined to point to strategic behavior by rational members as the most serious impediment to the successful provision of religious club goods. Under the auspices of the prisoner's dilemma, and the free riding it should entail, most, if not all, groups and factions beyond a minimal size should fail to corral the efforts of their members (Olson 1965). Various theories of the firm and other forms of collective action go to great lengths to demonstrate how such problems are overcome (Ostrom 2000; Williamson 2002). Religious groups typically operate without any wage or contract structure, and still produce a club good largely dependent on members whose efforts are, at best, difficult to monitor. Further, these groups often impose non-trivial costs on their members. At first glance, this imposition of costs would appear to present an exception to the first law of demand. Iannaccone's (1992) theory of sacrifice and stigma reconciles this apparent tension between theory and reality, positing that these costs, in the form of forsaken extra-group (secular) opportunities, make it possible for groups to overcome problems of free riding on behalf its members. The quality of a good changes with increasing costs, causing a shift in demand that, observed in a static/homogeneous good context, might give the impression of an upward sloping demand curve. Rather, greater sacrifice requirements are increasing the quality of the club by serving as a clever screening mechanism and aligning member incentives with group preferences. Acceptance and application of this theory has become increasingly numerous in the scientific study of religion, within both sociology and economics (Berman 2000; Stark and Finke 2000; Berman 2003; Keister 2003; Cosgel and Minkler 2004; McBride 2008).

The sacrifice and stigma theory is built using a representative agent, with groups comprised of agents homogenous across all attributes, most notably income. The theory predicts that an agent may experience increasing utility in the face of an increasing cost of secular, non-group activities. These benefits are monotonically increasing in both directions from a global minimum ($f'' > 0$). A group will maximize the utility of its members either by enforcing either complete sacrifice of all secular activities or no sacrifice at all. Applying this construct to different types of representative agents, as well as varying any and all of the model parameters, leads to varying conclusions as to which corner solution is optimal, but never to the possibility of a utility maximizing amount of sacrifice within the intermediate values.

As with most simplifying assumptions, the representative agent assumption is empirically false and incredibly useful. The model outcome that religious groups can only successfully avoid crippling free riding by demanding absolute sacrifice of all things secular is, of course, false as well. In extending and reconstructing the Iannaccone model of religion to allow for agent heterogeneity, we find that intra-group wage differentials are the key to allowing for the viability and dominance of moderate religious groups within the model. This emergence is a direct result of the ability of members to free ride off the greater efforts of members of the congregation with relatively lower opportunity costs of time. Small sacrifices serve as a barrier to entry for higher wage agents, which staggered across a religious landscape serve to segregate a population's religious groups by wages and preferences. Members find it utility maximizing to stay within a particular group due to their capacity to free ride off of fellow members and their unwillingness to defect to a group that demands greater sacrifice. Thus the sacrifice mechanisms works both to realign incentives within the group, as demonstrated by Iannaccone (1992), and as a screening device for prospective members with unobservable attributes, as used by Berman(2000; 2003) and Iannaccone (1994).

2 Survey Data

Religious groups in the United States come in tremendous variety, but there are few that would be considered "extreme" in their demands of members, on both an absolute scale and relative to the distribution of religious groups. Instead, we find a spectrum largely dominated by groups requiring contributions of wealth, time, and energy rarely more than 10% of an individual's "full" income. We define full income in the Beckerian sense as the individual's maximum productive capacity (Becker 1965).

The 2005 Baylor Religious Survey presents considerable evidence of the vitality and dominance of moderate religious groups in the United States. While it does not rule out the existence of extremist groups, it does demonstrate their limited profile. Using results from questions regarding time spent attending religious services (mass), volunteering for their congregation, and engaged in religious service activities, we create an approximation of time spent dedicated to religious groups during the last year. We use responses to questions regarding income and hours worked the previous week to impute a wage rate. This imputed wage rate is translated into a respondent's potential "full income" (Becker 1965). From this we calculated the fraction of a respondent's estimated full income that was dedicated to religious activity associated directly with his or her congregation, R_{FRAC} (see Appendix A for a breakdown of survey questions used and the imputation of values). Summary statistics of the survey variables used as inputs into the calculation of R_{FRAC} and a description of response format in the Baylor survey are included in Table 1. In the Baylor survey data, the mean respondent R_{FRAC} was 3.56% with a standard deviation of XX (see Table 1). The interquartile range of R_{FRAC} is 0.05% to 5.2%, offering further evidence that the majority of respondents were members of religious groups that demanded commitments that we casually classify as "moderate."

[TABLE 1]

While there is no explicitly quantitative measure of "sacrifice and stigma" possible, there has been tremendous effort to categorize American religious denominations with regards to their level of "strictness," "sectarian-ness," "tension," or sacrifice (Johnson 1963; Stark and Bainbridge 1980; Iannaccone 1997; Steensland et al. 2000). In Figures 1 and 2 we can see that the fractions of full income and hours dedicated to congregational activity per year within the different respondent denominations corresponds nicely with the level of sectarian-ness generally associated with those groups.

[FIGURE 1]

[FIGURE 2]

At the upper end of the R_{FRAC} spectrum in Figure 1, we see find Pentecostals, Church of Latter Day Saints, Mennonites, and Church of Nazarene with mean responses correlating to 8% or greater fractions of full income committed to their congregations, with Church of the Nazarene identifiers topping the list, approaching 20%. Each of these groups are considering strict and, compared to most groups with lower mean values, more sectarian (Hoge 1979; Iannaccone 1994; Woodberry and Smith 1998). Closer to the middle of the pack we find self identified Methodists, Congregationalists, and Lutherans, all hovering around mean R_{FRAC} of 5%, all groups which most scholars would comfortably identify as moderate (Smith 1990; Steensland, Park et al. 2000). At the lower end of the spectrum we find Unitarian Universalists, Jews (broadly identified), and Catholics.[1]

The broader point to be taken from Figures 1 and 2 is the prominence of groups commonly associated with the American religious mainstream and the relatively moderate demands they place on their members. The American religious groups represented in the sample of survey respondents are allowing members to retain the bulk of their resources, and remain highly productive outside of their groups. At the same time, 4% of full income is a non-trivial fraction of an individual's productive capacity. Moderate groups would appear to be not only viable in the United States, but in fact the dominant strand in the religious mainstream.

3 The Multi-Agent Computational Model

We construct our computational model with mathematical underpinnings explicitly based on Iannaccone's original model. Adapting the original model to accommodate a heterogeneous, multi-agent framework allows us to test the implications of the club model of religion for the different types of groups vying for members in a religious marketplace. In this paper we specifically explore the viability of moderate sacrifice groups, their ability to recruit members, and the level of commitment that moderate sacrifice engenders amongst its members. Given a distribution of agents with

[1] We also find a handful of religious groups, some not generally associated with moderate requirements, that do not fit the mold of traditional western congregational religion, such as Buddhism and Islam. The commitment of these individuals is likely not properly represented by our metrics.

heterogeneous wages, we will also investigate the distributions of wages within groups and average wages across groups.

The club model of religion begins with the premise that agents internally produce their own utility. This production relies on two inputs which are similarly produced by the individual in their secular (private) endeavors and their religious (group) endeavors. Both secular and religious production require commitments of time and money, each of which is limited in supply. Time endowments are homogenous across individuals, while money is a function of wages that are heterogeneous and exogenously assigned across the population. What makes the production of the religious input unique is the interdependence of religious production with other members of the group. This interdependence invites members to free ride – to be a member of the group and benefit from the religious production of other members while in turn neglecting her own religious production. Iannaccone's crucial insight was that the imposition of costly sacrifice and stigma requirements could mitigate the free rider problem, resulting in rational members whose choice to engage in more religious production increased not just their own utility, but the utility of all other members.

Individuals are heterogeneous in their wages, but identical in their basic preferences. Similarly, religious groups are heterogeneous in their sacrifice and stigma requirements, but are identical in their capacity to produce the religious "club good." What can in turn emerge is a religious economy within which some groups succeed in attracting members and others fail. Within this economy, individuals will decide how best to invest their scarce resources – whether to produce their own utility by allocating their time and money to secular endeavors or to their chosen religious group. A spectrum of agent choices will also emerge, including the secular independent, the devout group member, and everything in between.

In the model constructed, each agent produces her own utility with a constant elasticity of substitution (CES) production function, with inputs of a secular, private good *S*, and a religious, club good *K*, preference parameters d_S and d_K, and a substitution parameter β. *S* and *K* are classic "Z-good" arguments in the utility function (Stigler and Becker 1977). *K* is produced with a Cobb-Douglas production function with constant returns to scale and inputs R_i, the individual's contribution, and Q_g, the "quality" of the other group members' contributions, with output elasticity parameters α and 1- α.

(1)
$$\begin{aligned} U_i &= (d_S S_i^{\beta} + d_K K_i^{\beta})^{1/\beta} \\ K_i &= (R_i^{\alpha} Q_g^{1-\alpha}) \end{aligned}$$

The group quality input, $Q_{i,g}$ is defined as a function of the average input *R* across agent *i*'s neighbors ($j \neq i$), a scalar $s > 0$, and the number of agent *i*'s neighbors, n_g, that are members of the group, *g*, being evaluated.

(2)
$$Q_{i,g} = R_{g,j \neq i} \cdot s(1 - \frac{1}{1+n_g}))$$

$Q_{i,g}$ is strictly increasing in n_g, with diminishing marginal returns $(Q' > 0, Q'' < 0)$. This formulation of $Q_{i,g}$ is an important mathematical change from the original model. The original model hinges on a Nash-Equilibrium assumption ($R_i = R_{j \neq i}$), creating a prisoner's dilemma. In our model, agents are able to observe local agent behavior different from their own, and in turn inform their own decision-making. As such, $R_i = R_{j \neq i}$ no longer necessarily holds true and the model ceases to have a closed-form equilibrium solution.[2] Because we are operating in a computer-aided framework, however, we are less dependent on finding closed-form solutions.[3] The utility function, for any given value of $Q_{i,g}$, contains only a single, global maximum, which allows the luxury of employing the relatively simple golden mean search optimization algorithm (Press 2002).

S and *R* are both Cobb-Douglas produced with inputs of goods, x_S and x_R (prices p_S and p_R); and time v_S and v_R; input elasticity parameters *a* and *b*; and production capacity parameters A_S and A_R. A_S is the dimension in which group sacrifice is implemented.

(3)
$$S_i = A_S(x_{i,S}^{a} v_{i,S}^{1-a})$$
$$R_i = A_R(x_{i,R}^{b} v_{i,R}^{1-b})$$

Agent's are exogenously endowed with a heterogeneous wage rate, w_i, and a uniform time endowment $V = v_{i,S} + v_{i,R}$. Using the envelope theorem, we can construct shadow prices π_R and π_S.[4] With agent specific shadow prices established, agent choice is an exercise in standard optimization constrained by the agents' exogenously endowed full income $(p_S x_{i,S} + p_R x_{i,R}) + (w_i v_{i,S} + w_i v_{i,R}) \leq I_i$ (Becker 1965), defined as the value of goods purchased and wages forgone to time invested, where *w* is the agent's wage rate and p_S and p_R are the prices for secular (x_S) and religious goods (x_R) .

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[2] The computational model generates outcomes equivalent to the Nash Equilibrium outcome of Iannaccone's original model when constrained to a representative agent. The implied two-group outcome possibility can also be generated if two agent types are employed.

[3] The model is written in Java 1.5.1 using the MASON agent modeling library (Luke et al. 2005).

[4]
$$\pi_S = p_S \;\; \partial x_S^* / \partial S \;\; + w_i \;\; \partial v_S^* / \partial S \;\; = 1/A_S \;\; p_S \;\; aw_i/(1-a)p_S \;\;{}^{1-a} + w_i \;\; aw_i/(1-a)p_S \;\;{}^{-a}$$
$$\pi_R = p_R \;\; \partial x_R^* / \partial S \;\; + w_i \;\; \partial v_R^* / \partial S \;\; = 1/A_R \;\; p_R \;\; bw_i/(1-b)p_R \;\;{}^{1-b} + w_i \;\; bw_i/(1-b)p_R \;\;{}^{-b}$$

In evaluating Q_i^g, agent *i* is evaluating agents currently occupying patches in her neighborhood who are members of group $g \in G$ where $G = \ 0,1,2...m$. The groups in set G are differentiated along required member sacrifice in private productivity parameter A_S^g , where:

$$A_S^g \begin{cases} 1-0.9^{(m-g)}+\varepsilon & \text{if} \quad g>0 \\ 1 & \phantom{\text{if}} \quad g=0 \end{cases} \tag{6}$$

The sacrifice that a group enforces comes at the expense of A_S^g, where the first group (g = 0) offers member private productivity parameter A_S^0 = 1 (no sacrifice) and the final group requires $A_S^{G-1} = \varepsilon$ (complete sacrifice, where ε is defined as an arbitrarily small value to prevent division by zero). The resultant sacrifice is $1 - A_S^g$.[5]

The computational model exists as a two dimensional lattice (Figure 3) not unlike a checkerboard, on which agents occupy spaces identified here as “patches.” Agents are one to a patch, and have a set of eight neighboring patches (four adjacent and four on the diagonals) whose occupants make up their “neighborhood.” Within this spatial context agents engage in local (as opposed to global) optimization, choosing the group and personal investment in club production that maximizes utility in their own unique local context. Given that each agent holds a unique set of coordinates and neighbors during any time step of the model, the spatial construct represents an important source of agent heterogeneity in the model.

[FIGURE 3]

[5] Different bases were tested for the sacrifice function. As the number of groups is increased, the model becomes more fine grained, but at the cost of speed and ease of data collection and analysis. The formula employed allows for finer grained analysis at the lower end of the sacrifice spectrum and sufficient variety at the higher end, while limiting the model to what proved to be a tractable number of groups.

3.1 Model Steps

A run of the model consists of initialization followed by a set number of time steps, summarized in the following time structure:

Step [t = 0] Initialization. The model creates and places agents randomly, one per patch. Agents are heterogeneous across wages, pulling random values from a lognormal wage distribution. Agents are randomly assigned an initial group from a set of *G* different groups. Upon their creation, agents optimize their values of *R* and *S* as a function of their wage and the sacrifice required by their initial group in an iterated Cournot-Nash solution to a game that the agent plays against herself. This is the only time that agents will act without knowledge of their neighbors.

Step [t > 0]. Each model step consists of shuffling the order of agents and one at a time progressing through their ranks. When it is agent *i*'s turn, she will evaluate $Q_{i,g}$ for each prospective group, *g*, that is represented in her neighborhood[6]. The optimal *R* and *S* are determined for each potential group, with the agent joining the utility maximizing group. The choice of group for an agent is a function of her wage, each group's respective sacrifice demanded, the quantity and commitment (in terms of *R)* of the representatives of each group in her neighborhood, which is in turn a function of their wages and the decisions made by their neighbors, and so forth. The actual emergent collection of groups is a property of a, perhaps, surprisingly complex process for a model rooted in a standard CES structure.

4 Experiment and Results

[6] The set of groups considered always includes the zero-sacrifice group, regardless of whether a member resides in the agent's neighborhood.

The model was set to run with a 33 by 33 lattice, filled with 1,089 agents, divided amongst 60 initial groups. The income distribution is lognormal with a shape parameter of one. All preference parameters are set to unity. The scalar multiplier, *s,* in the calculation of *Q* is set to 1.25.[7] Parameter assumptions are summarized in Table 2.

[TABLE 2]

The β and mean wage, μ, are the key parameters explored in this chapter. The experiment varies the β in increments of 0.1 between 0.4 and 0.9.[8] The mean wage is varied in units of 0.5 between 0.5 and 4.0. Each combination of β and μ is run 50 times, with 100 turns constituting a run. In the course of a turn, each of the 1,089 agents is activated in random order. When an agent is activated, she surveys her local neighborhood and chooses the utility maximizing group.

The sacrifice and stigma theory predicts that groups require seemingly wasteful sacrifice as a means to solving the free riding problem. It is to be expected that as sacrifice increases, so does the commitment of resources to the club by members. This is exactly what happens in the simulation model, as seen in Figure 4, with the fraction of full income dedicated by agents plotted against the level of sacrifice they accept from their group.

[FIGURE 4]

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[7] The scale parameter, other parameters held constant, shifts the population profile towards the club good, and in terms of sacrifice, towards the more extreme. As s increases, the size of K relative to S increases for all values of Q. The rewards to sacrifice are, in turn, increasing with s. This linearly impacts the commitment profile of the population, but does not meaningfully impact the results of the model discussed here when constrained to a reasonable range of values. Future work related to the success of "mega churches" may wish to explore the impact of the scale parameter in other contexts.

[8]As Iannaccone notes in the original article, the sacrifice mechanism only succeeds can only be successful if $\beta > \alpha$.

The sacrifice profile of a population of agents is represented in Figure 5 as a bar chart of membership size across groups. Each bar represents $\log(n_g)$ of a different group, in increasing order of sacrifice required by the group $g \in [0,59]$. The actual sacrifice level of the group is between 0 and 100%, shown in the lower portion of Figure 5, increasing in accord with Equation 6. Figure 5 shows the overlapping results of a series of runs with identical beta parameterization, β = 0.7, but a range of mean population full incomes, $\mu \cdot T = [8,64]$ in 8 unit increments. The entire range of sacrifice levels allow for groups that are potentially viable in the population.

[FIGURE 5]

Figures 6 and7 allow a comparison of different mean population incomes, again noting the breadth of sacrifice levels that make for viable groups. In the relatively high population income simulations (Figure 6), moderate religion is robust across all substitutability parameter values (β). The appeal of liberal, low sacrifice, groups increases as β decreases, especially for liberal groups whose sacrifice requirements are greater than zero. In simulations with lower average population incomes, however, moderate religion remains viable, but only so long as substitutability remains sufficiently weak (Figure 7). When β = 0.9, meaning private and group activities are nearly perfect substitutes for one another, we see results analogous to what would be derivative of the original Iannaccone model. Specifically, the only groups that are successful are those demanding either very large sacrifice or none at all. Regardless of income, greater substitutability polarizes the population, pushing them towards the highest and lowest sacrifice groups. Weak substitutability is sufficient, and nearly necessary for moderate sacrifice groups to remain viable over time within a population of heterogeneous agents. Relatively wealthier income distributions favor moderation, but high income is neither necessary nor sufficient for the viability of moderate sacrifice groups.

[FIGURE 6]

[FIGURE 7]

5 How Moderate Groups Persist

In Iannaccone's original model, utility monotonically increases in both directions from the global minimum.[9] Only the corner solutions hold the potential for optimality, leaving the individual with the choice of either sacrificing all of her secular productive capacity or none of it. In our multi-agent model, however, we find that a continuum of groups is possible. While the continuum from zero to 100% sacrifice may have significant discontinuities, depending on the size of the population simulated and the parameters governing the model, the viability of groups demanding intermediate levels of sacrifice is evident. How is this possible given the underlying mathematics governing the model?

The key is the heterogeneity of wages across agents. Put simply, agents who earn lower wages place a greater fraction of their productive capacity into time intensive religious endeavors from which other group members benefit.[10] Groups are only viable with members $n_g > 1$ (with the exception of the zero sacrifice "group" which could

[9] For a visual reference, see Figure 11, which is comparable to the appropriate figure from the numerical example in the original Iannaccone paper, where $\beta = 0.8$, though there the x axis was the log of the ratio of shadow prices, where in figure 11 it is the log of the productivity parameter.

[10] McBride(2007), offers alternative analysis from the point of view of the group, who themselves stand to benefit from free-riders who can build social capital in the group and contribute more in the future. The group he is modeling, the Church of Latter Day Saints, is a great example of a relatively high, but still moderate, sacrifice group that continues to thrive and grow.

feasibly be made up of a single secular loner, and as such the number of active groups will be less than or equal to $\frac{N}{2}-1$. Population wages are pulled from a continuous distribution and as such each group will be composed of members with unique, though perhaps similar, wages. When group members earn a variety of wage rates, an individual agent has incentive to find a group whose members are *relatively poorer* than she is. In the classic free rider's gambit, she wants fellow members who put more into the group than she does.

There are limits to this incentive, however. Dependent on the parameterization of the output elasticities of time and goods to the production of *S* and *R*, while lower wage agents have a *comparative* advantage (equation 4) in the production of religious group goods, higher wage agents have an *absolute* advantage (equation 5) in the production of all goods. In equations 4 and 5, R^*, S^* represent the optimal choices of *R* and *S*, and $R^{\max}$ is the maximum value of *R* possible.

(4) $$\text{if } b < a \text{ and } w_i > w_j \text{ then } \frac{R_i^*}{R_i^* + S_i^*} < \frac{R_j^*}{R_j^* + S_j^*}$$

(5) $$\text{if } b > 0 \text{ and } w_i > w_j \text{ then } R_i^{\max} > R_j^{\max}$$

Agents will only desire relatively poorer fellow members so long as their comparative advantage outweighs their absolute disadvantage such that $R_j^* \geq R_i^*$.

The potential for utility enhancing moderate sacrifice can be demonstrated through a numerical exercise. This exercise revolves around a two-player game of differing wages, informed by various results from the simulations of our multi-agent model, and can illustrate how model conditions, and the interaction mechanics they underpin, allow for utility maximizing moderate sacrifice. Moderate sacrifice remains viable so long as the returns to increased sacrifice are positive, specifically so long as

$$\frac{dU_i}{dR_j}\overset{(+)}{\frac{dR_j}{d\pi_S}}+\frac{dU_i}{dS_i}\overset{(-)}{\frac{dS_i}{d\pi_S}}-\frac{dU_i}{dR_i}\overset{(+)}{\frac{dR_i}{d\pi_S}}>0 \ .$$

We here recast the original model in terms of a group with two members, agents *i* and *j*.

(7) $$U_i = (b_S S_i^{\beta} + b_K K_i^{\beta})^{1/\beta}$$

(8) $$K_i = (R_i^{\alpha} R_j^{1-\alpha})$$

Q_i has here been replaced with R_j, the religious group production of the sole other member. Shadow prices are calculated as before (see footnote 3). In the numerical example of Iannaccone's original paper, Marshallian demands are derived with standard optimization and a Nash Equilibrium assumption,[11] which we employ here.

Agent wages within our earlier simulation model are pulled from a lognormal distribution. Running the model for a 100 time steps allows agents to find their respective optimal groups. As expected, higher wage agents are attracted to lower sacrifice groups, and vice versa. This sorting process, however, results in groups whose within group median and maximum wages are more heavily skewed than population wages,

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[11]
$$R^{e}(\pi_S,\pi_R,\beta,I,\alpha)= \ (\alpha^{1/\beta})^{\beta/1-\beta}\pi_R^{\ 1/\beta-1}I \ \ / \ \ \pi_S^{\ \beta/\beta-1}+(\pi_R/\alpha^{1/\beta})^{\beta/\beta-1}$$
$$S^{e}(\pi_S,\pi_R,\beta,I,\alpha)= \ \pi_S^{\ 1/\beta-1}I \ \ / \ \ \pi_S^{\ \beta/\beta-1}+(\pi_R/\alpha^{1/\beta})^{\beta/\beta-1}$$

demonstrating possible power law characteristics likely derivative, at least in part, from the exogenously set scaling of sacrifice across groups (see Figure 8).

[FIGURE 8]

We factor this skewing of group maximum wages into the two player game by making agent j's income a a function of agent i's income and the sacrifice level of their two-player group, such that $w_j = w_i^{\phi} A^{\theta}$. We chose the parameters of this function by returning to the simulation data generating in the previous experiments. Agent *i's* preferred outcome is to find a group with the lowest sacrifice level *within which he has the highest wage*. Agent *i*, whom we can think of as an "average" agent, wants to be give up as little as possible, but have everyone else contribute more to the group than she does. Our average agent *i* wants to predict what the highest wage (other than her) in each group will be, and hopefully join a group where she can be the highest wage agent. Using the simulation data, we can regress highest wage of members of each group. $\mathbf{max}(w_i)_g$, on the productivity factor of the group, A_g (where $1 - A_g$ is the sacrifice requirement), and the mean wage of the entire population, μ.

(9) $$\mathbf{max}(w_i)_g = \beta_1\mu + \beta_2 A_g + \varepsilon_g$$

What we expect is that the maximum wage of the group will be increasing with the productivity parameter (i.e. low sacrifice groups will have higher maximum wages) and increasing with the mean wage of the population, and that is exactly what we find.

[Table 3]

We use these results, seen in Table 3, to parameterize the numerical exercise: $w_j = w_i^2 A^{145}$.[12] We are reducing the rest of the group to only this other, **max**$(w_i)_g$ agent. Treating agent *i* as an “average” agent, as she evaluates groups she will find her counterpart in the group, agent *j*, will have 1) a much higher wage in lower sacrifice groups, and thus free ride too much himself 2) a much lower wage in the high sacrifice groups, and thus will bring too little productive capacity to the table, but c) have potentially a wage that is “just right,” contributing more to the group than she, agent *i*, will . Put differently, R_j will be increasing with small amounts of sacrifice, as *j*’s comparative advantage in R dominates his declining total productive capacity. Eventually, however, the early returns to R_j from sacrifice diminish as agent *j*’s comparative advantage is outweighed by his absolute productive disadvantage derivative of his lower income. The partner agents from higher sacrifice groups are actually producing a lower values of R_j, while giving a higher percentage of their full income, are contributing a lower total quantity of R_j. This takes the form of an observable local maximum of R_j at a value of A_g that is less than one in Figure 9.

[FIGURE 9]

[12] We are playing a bit fast and loose with the earlier Nash equilibrium assumption made by agents. The simple Marshallian demand functions are derivable because the agent is assuming the other agent is identical to herself. In setting up wj(wi,A), we are setting up an interaction between two agents who are explicitly not identical. Agents in this exercise have demand functions that are derivable because they assume the other agent is identical, even though he is not. Agent rationality is thus heavily bounded, in that they are making decisions using themselves as the model of behavior expected in others, even though their model is false.

Derivative of the previous result, first, U_i is increasing with additional sacrifice early on, as the benefits from increasing group efforts of agent *j* outweigh utility lost to sacrificed secular productive capacity. Small sacrifices can attract a membership with greater commitment to the club good, in spite of a reduced aggregate productive capacity. This takes the form of an observable local maximum of U_i at a value of A_g that is less than one in Figure 10.

[FIGURE 10]

The importance of substitutability (β) can be understood in the context of the original model, the numerical computation of a two-player game, and the simple economics of substitutes and complements. In Figure 11 we see utility graphed over *A*, where *A* is the secular productivity factor which is manipulated as the sacrifice parameter; lower amounts of *A* correspond to higher π_S and greater sacrifice. The U-shape is familiar to those who have read Iannaccone's original paper; the utility producing mechanisms of two-man game are not substantially different from the representative agent model. What is important to note is the flattening shape of the function as β decreases, demonstrating that $\frac{{}^{d^2U}\!/\!{}_{d^2A}}{d\beta} > 0$ (see Fig. 11). This flattening out reduces the absolute value of $\overset{(-)}{\frac{dU_i}{dS_i}\frac{dS_i}{d\pi_S}}$, increasing the range over which $\overset{(+)}{\frac{dU_i}{dR_i}\frac{dR_i}{d\pi_S}}$ may be sufficiently large such that a positive return to a moderate amount of sacrifice is possible. Weaker substitutability reduces the secular opportunity cost of small moderate sacrifice.

[FIGURE 11]

The importance of the substitutability parameter is also realized in the production of religious capital by higher income agents. As β decreases, the substitutability of the two goods weakens and the complementarity strengthens. The minimum amount of religious production by the wealthiest agent increases as the efficient production of utility is dependent on increasingly more equitable amounts of its two “Z-good” inputs. The wealthier agent stands to gain more from the inclusion of religious group capital in her portfolio as the complementarity, however marginal, of S_i and K_i capital increases. As a result, the marginal reward to the poorer individual’s contribution is less burdened by the free riding of wealthier members as β decreases.

The sacrifice level stands as incentive for members to increase their production ratio of $R:S$ and for high wage agents, who are too great of a free-riding burden, to find their membership elsewhere. The sacrifice level serves as a one-way barrier keeping out higher income free riders. No barrier is needed to keep out lower income agents; they are always desired so long as negative returns to group scale are absent. The emergent group forms as wealthier agents find the sacrifice level prohibitive and poorer agents find the group’s religious production, derivative of their sacrifice demands and respective incomes, insufficient. Within these thresholds of income, a group finds its membership.

As β drops, the largest viable sacrifice percent diminishes. Increasing the complementarity of Z-goods will result in all agents contributing significant portions of their productive capacity to the club good, while simultaneously bearing a greater sensitivity to any loss in secular production. Complementarity runs in both directions –

lower β means the utility producing power of *K* is more dependent on relatively higher levels of *S*. These conditions lead to a substitutability "sweet spot" for any heterogeneous population. There exists a range of β values that lead to both the greater viability of moderate groups, and a greater range of sacrifice requirements that can support viable groups, much of which falls in the intermediary subset that would be considered moderate in magnitude and member commitment outcomes.

6 Conclusions

Liebman felt it was moderate, not extremist, religion that needed explanation. Iannaccone's sacrifice and stigma model presented a compelling solution to the paradox of voluntary sacrifice, but left unanswered the question how groups could survive not just in the moderate portion of the spectrum, but rather in the entire interval between zero and complete sacrifice; the intermediary where virtually all modern religious groups find themselves. It is only by relaxing the assumption of agent homogeneity that we can model the religious group as a social construct that can thrive by demanding a moderate amount of sacrifice.

The interacting mechanics of comparative and absolute advantage that allow moderate religion to persist emphasize the importance of not just mean wages, but the shape of the wage distribution. Within the model, wage inequality has the positive externality of moderating groups, providing an incentive to reduce demands of sacrifice in an effort to make the group more attractive to prospective members with greater financial means. This incentive to lure wealthier members, in light of evidence regarding intra-congregation skew in the distribution of member donations (Iannaccone 1997), has potential implications for the observed decline in strictness and commitment within many denominations that leads, possibly cyclically, to sectarian movements and schism (Montgomery 1996).

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Table 1. 2005 Baylor Religious Survey

	Mean (Std.Dev.)	Survey Scale [# of response options]	Conversion
Income	66,452 (52,815)	Range [7]	Integer
Hours worked last week	26.34 (21.70)	Integer	No
Full Income*	233,781 (437,911)	N/A	N/A
Tithe per Year (dollars)	1385.24 (2609.93)	Range [12]	Integer
Tithe as % of Income	2.61 % (5.06 %)	N/A	N/A
Hours Volunteering for Religious Group per Year	17.65 (38.15)	Range [5]	Integer
Hours is Religious Activities per Year (10 items)	129.43 (228.22)	Range [4]	Integer
Hours at Religious Service/Mass per Year	27.85 (31.41)	Range [9]	Integer
Religious Fraction of Full Income+	3.56 % (5.37%)	N/A	N/A

Standard deviations in parentheses *Full Income is calculated by extrapolating hours worked last year to calculate an hourly wage. This wage is then applied to a 16 hour work day, and multiplied by 365. + Religious Fraction of Full Income is the wage value of time spent in religious services, volunteering to the congregation, and religious activities plus tithing, as a fraction of full income. See Appendix A for data imputations.

Table 2. Model Parameters

Parameter	Related Function	Value
d_s, d_k	$U_i = (d_S S_i^{\beta} + d_K K_i^{\beta})^{1/\beta}$	1
s	$Q = R_{j \neq i} \cdot s(1 - \frac{1}{1+n}))$	1.25
α	$K_i = (R^{\alpha} Q^{1-\alpha})$	0.3
a	$S = A_S(x_S^a t_S^{1-a})$	0.7
b	$R = A_R(x_R^b t_R^{1-b})$	0.3
G	(number of Groups)	60
p_S, p_R	(prices of goods)	1

Table 3. Log-Linear regression of Maximum Wages in Moderate Groups†

	Group Maximum Wage
Log μ	1.991 (0.012)
Log A_g	145.143 (2.629)
R^2	0.9325

†N = 3751**.** The the regression is run over the subset of "moderate" groups: g <= 20; β = 0.7.

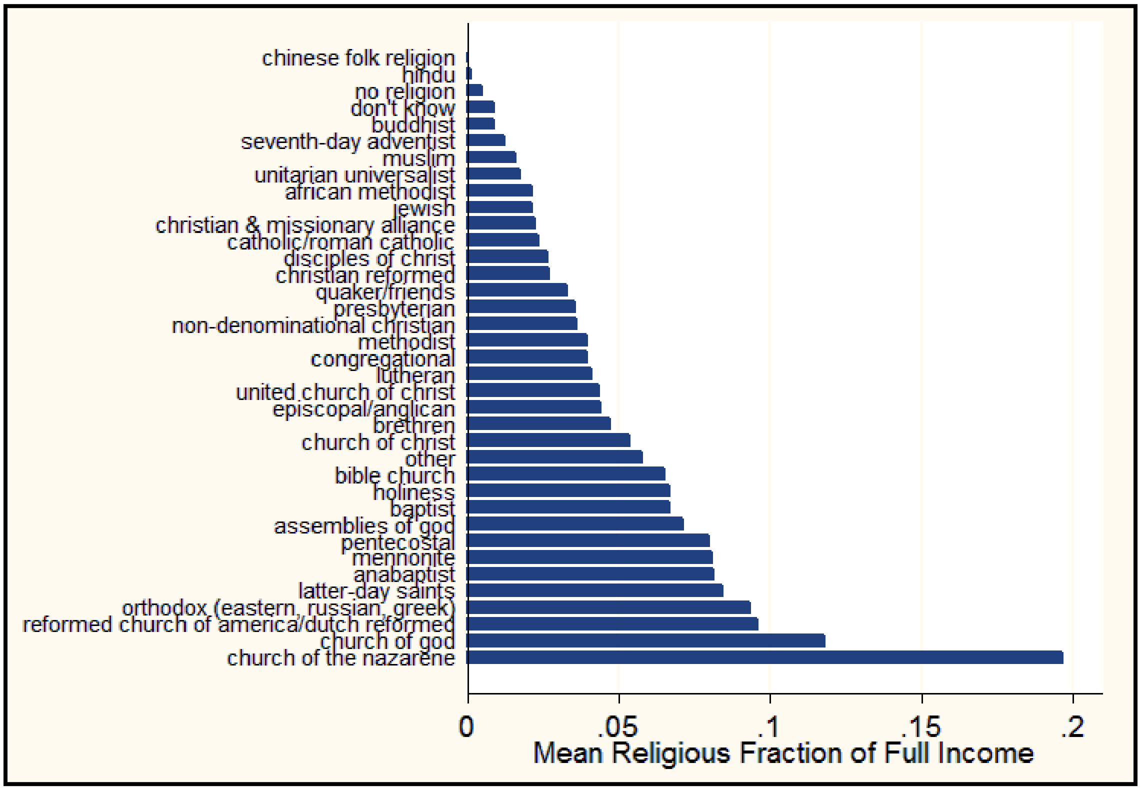

Figure 1. Mean Fraction of full income dedicated to the respondent's religious congregation, organized by denomination. Denominations with an insufficient number of wage and/or hours worked responses are excluded (see Figure 2 for commitment in hours for additional denominations). See Appendix A for data imputations.

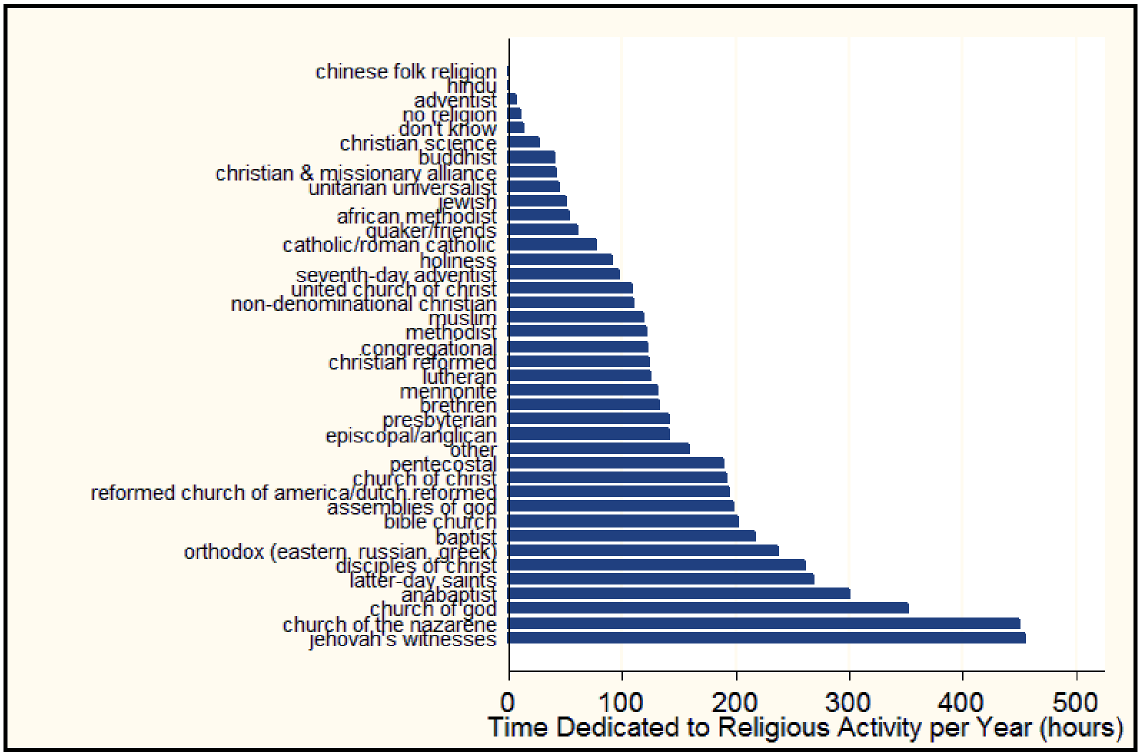

Figure 2. Mean hours dedicated to the respondents congregation, by denomination. See Appendix A for data imputations.

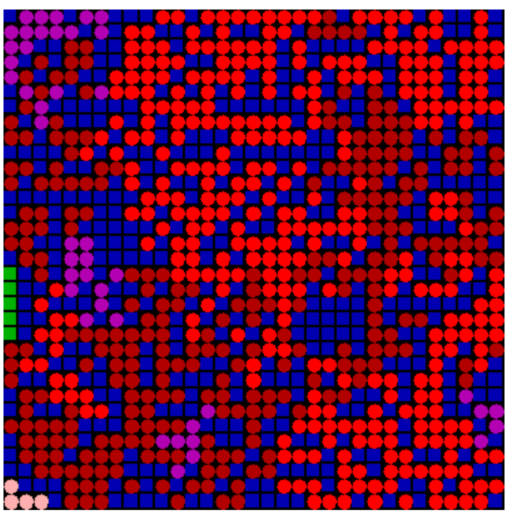

Figure 3 33 by 33 lattice

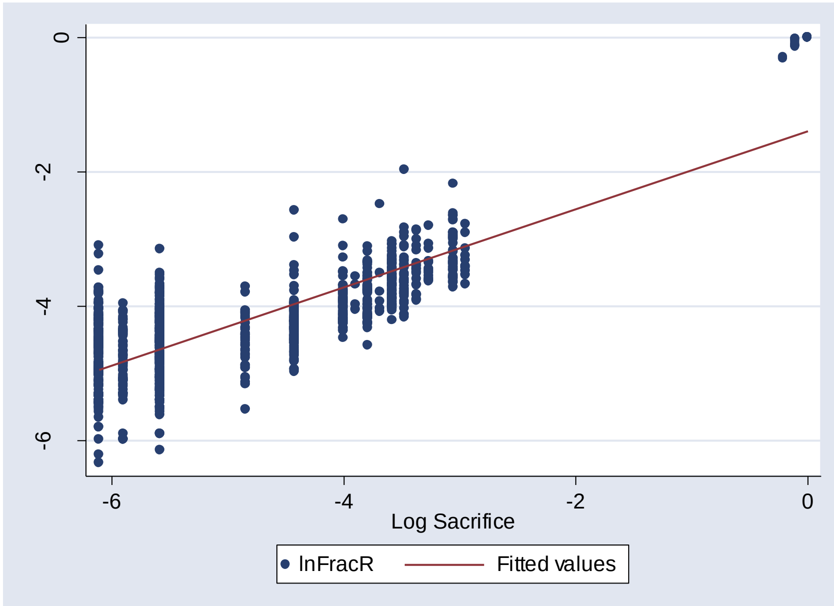

Figure 4. Log R as Fraction of Full Income over Log Sacrifice

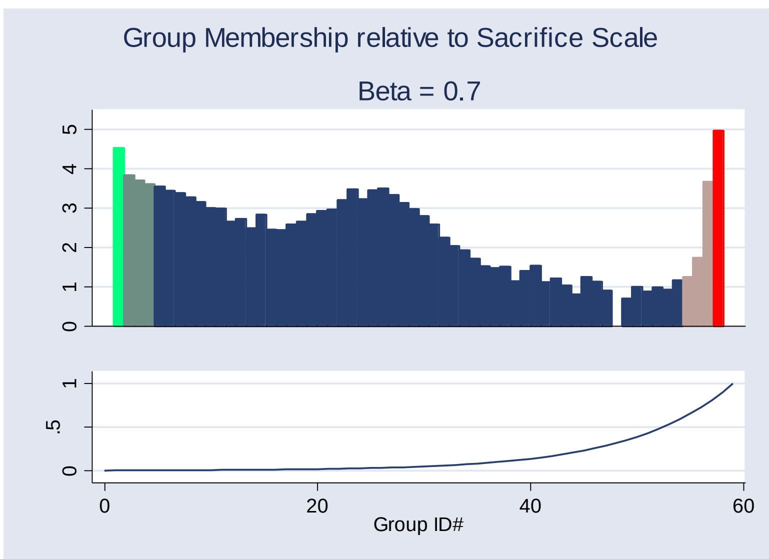

Figure 5. Distribution of Members across Groups, Sacrifice Scale by Group

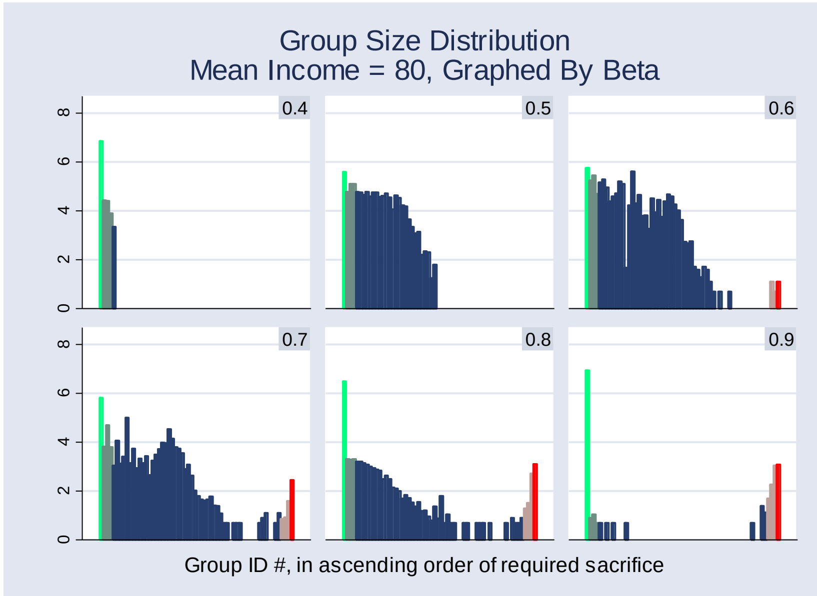

Figure 6. Distribution across beta – High income By Group

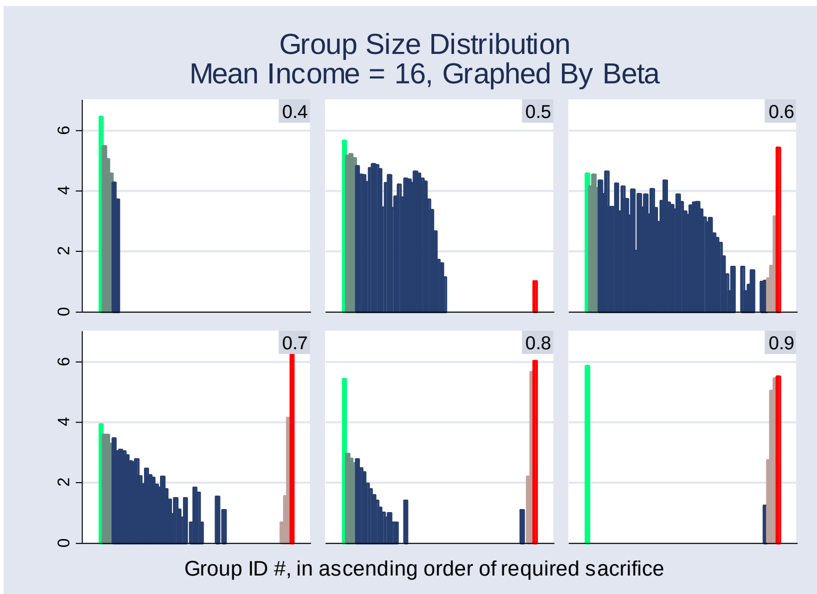

Figure 7. Low income Population, Distribution across beta, By Group

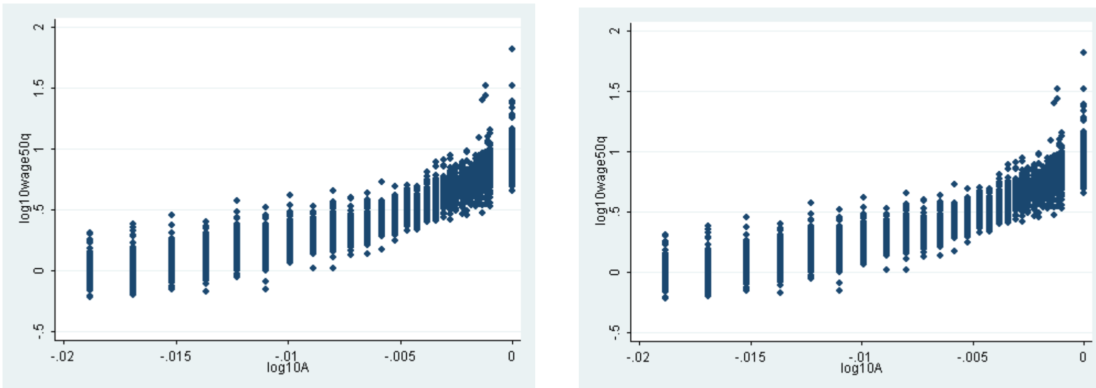

Figure 8. Log (base 10) group median and maximum wages by log (base 10) A_g. β=0.7; N=10,000.

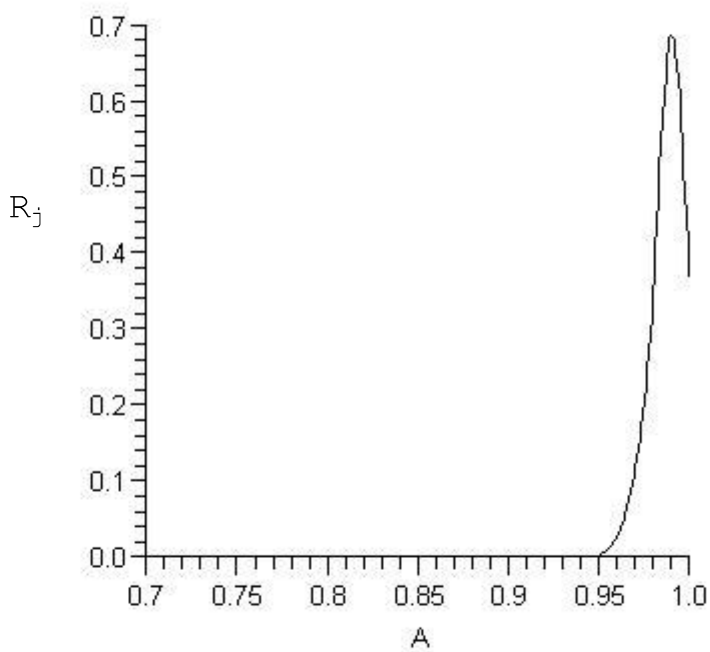

Figure 9. Production of R by agent i relative to the club productivity parameter A. Lower values of A correspond to larger sacrifice levels required by the group. β=0.7; w_i=4.

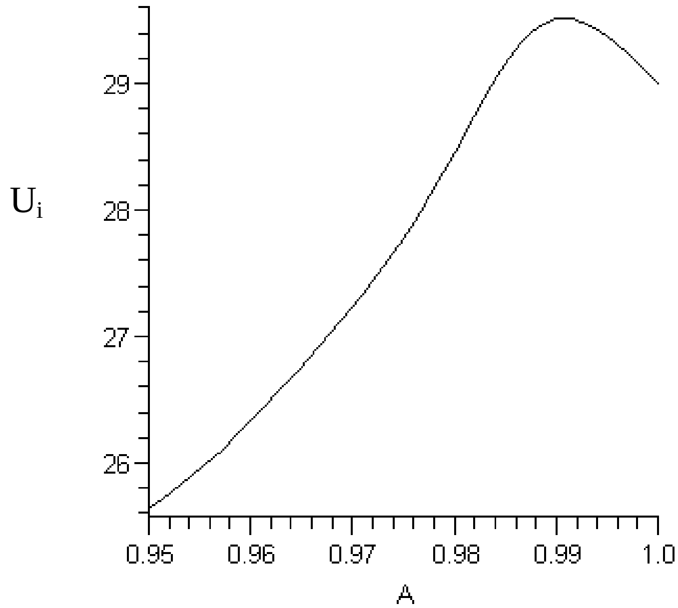

Figure 10. Utility for agent *i* relative to the club productivity parameter A. Lower values of A correspond to larger sacrifice levels required by the group. β=0.7; w_i=4.

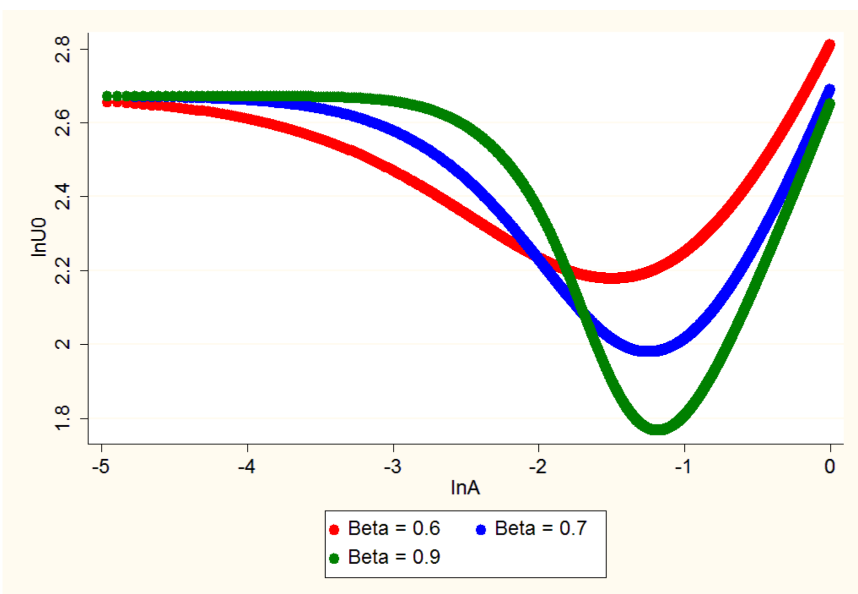

Figure 11. Log Utility over Log A, for various values of β.

Appendix A

Imputation of Religious Commitment Variables from the Baylor 2005 Survey

1) Income (per year), using categorical responses to Question 60

Income = 5000 if Question 60 = 1
Income = 15000 if Question 60 = 2
Income = 27500 if Question 60 = 3
Income = 42500 if Question 60 = 4
Income = 75000 if Question 60 = 5
Income = 125000 if Question 60 = 6
Income = 200000 if Question 60 = 7

2) Wage (per hour)

$$\text{Wage} = \frac{\text{Income}}{\text{Hours worked last week} \cdot 52}$$

Where "hours worked last week" were numerical responses to Question 62.

3) Tithe (per year) using categorical responses to Question 11

Tithe = 0 if Question 11=.
Tithe = 250 if Question 11=1
Tithe = 750 if Question 11=2
Tithe = 1500 if Question 11=3
Tithe = 2500 if Question 11=4
Tithe = 3500 if Question 11=5
Tithe = 4500 if Question 11=6
Tithe = 5500 if Question 11=7
Tithe = 6500 if Question 11=8
Tithe = 7500 if Question 11=9
Tithe = 8500 if Question 11=10
Tithe = 9500 if Question 11=11
Tithe = 15000 if Question 11=12

4) Service Time (per year) using categorical responses to Question 5

Service time = 0 if Question 5 = 1
Service time = 1 if Question 5 = 2
Service time = 2 if Question 5 = 3
Service time = 6 if Question 5 = 4
Service time = 12 if Question 5 = 5
Service time = 30 if Question 5 = 6
Service time = 44 if Question 5 = 7
Service time = 52 if Question 5 = 8

Service time = 104 if Question 5 = 9

5) Religious Activities Time

Question 14 a through j, religious activities (per month), where “religious time” is the sum of responses to 14 a through j.

Activity Time a = 0 if Question 14a = 1
Activity Time a = 3 if Question 14a = 2
Activity Time a = 7 if Question 14a = 3
Activity Time a = 15 if Question 14a = 4

6) Volunteering for the Church (per Year)

Volunteering = 0 if Question 48c =1
Volunteering = 18 if Question 48c =2
Volunteering = 42 if Question 48c =3
Volunteering = 90 if Question 48c =4
Volunteering = 180 if Question 48c =5

These imputed factors allow for the following calculation:

7) $$\text{Religious time} = \text{Volunteering} + (\text{Activity time} \cdot 12) + \text{Service time}$$

8) $$\text{Religious fraction} = \frac{(\text{Religious time} \cdot \text{Wage}) + \text{Tithe}}{\text{Full Income}}$$