Towson University
Department of Economics
Working Paper Series

Working Paper No. 2009-04

Technology Shocks and Business Cycle: An Analysis Based on the Malmquist Index

By Mark J. Lee

December 8, 2009

Technology Shocks and Business Cycle: An Analysis Based on the Malmquist Index

Mark J. Lee*
Department of Economics
Towson University

December 8, 2009

Abstract

The goal of this study examines the quantitative implications of the Malmquist index in a standard Real Business Cycle (RBC) model as a measure of technology shock. To achieve this, the paper first investigates the empirical validity of the equivalence proposition on the two technology shock measures: a relatively new Malmquist Index and the predominant Solow residual. On the basis of permutation tests, this paper shows the observational equivalence of the two measures. Then, the role of technology shock measured by the Malmquist index in the RBC model is examined. The study uncovers that the RBC model with the Malmquist index successfully replicates the stylized U.S. business cycle features documented in the existing literature. Finally, this paper discusses potential benefits of the Malmquist index in the business cycle studies.

JEL Classification: E32; O47

Keywords :Malmquist Index; Observational Equivalence; Solow Residual; RBC Model; Aggregate Technology Shocks

*I would like to thank Michael Boskin, Lawrence Lau, Kangoh Lee, Dirk Krueger, Tony Braun, Shawna Grosskopf, and session participants of 2006 NAPW in New York for their helpful comments on an earlier and this version of the paper. I also would like to acknowledge the hospitality of the Center for International Research on the Japanese Economy (CIRJE) at the University of Tokyo, where part of this research was conducted. Finally, I am especially grateful to Yasuyuki Sawada for his invaluable suggestions. I am solely responsible for all remaining errors. Mailing Address: Department of economics, Towson University, 8000 York Road, Towson, MD 21252, U.S.A. Tel: +1 (410) 704-2191; Fax: +1 (410) 704-3424. E-mail: mlee@towson.edu

1 Introduction

Most business cycle models predominantly use the Solow residual as a proxy for the measured aggregate technology shock. While an alternative measure, the Malmquist index, has been proved equivalent to the Solow residual by Caves, Christensen, and Diewert (1982) and used in many micro productivity studies, existing macro business cycle studies have somehow overlooked the Malmquist index.[1] Indeed, there are very few business cycle studies based on the Malmquist index, so the implications of such a measure are largely unknown. The goal of this paper is to fill this gap. In this paper, the implications of technology shock measured by the Malmquist index on the business cycle analysis are studied. In particular, the Malmquist index is presented as an alternative proxy for the measured technology shock and its quantitative effectiveness in a standard RBC model is evaluated. By doing so, this paper attempts to open a new area of application of the Malmquist index in macroeconomics, which might be one contribution of this paper.[2]

To appeal to the majority of business cycle researchers who are unfamiliar with the Malmquist index, this paper first provides a systematic comparison of the two measures by showing the *observational equivalence*[3] of the Malmquist index and the Solow residual, and then uses the Malmquist index when the quantitative effectiveness of a RBC model is assessed. The findings suggest that the Malmquist index is equally good as the Solow residual. Finally, this paper discusses potential benefits of the Malmquist index.

This paper contributes to the literature by (1) empirically comparing two aggregate technology shock measures, and (2) then studying the quantitative effectiveness of the Malmquist

[1]This study uses the terms *the Solow residual* and *the Malmquist index* to refer to aggregate technology *changes*, not levels. The Solow residual is defined as changes in the Tönquvist indices, and the Malmquist index, as the geometric mean of two Malmquist productivity indices.

[2]The first serious attempt was made by Färe, Grosskopf, Norris, and Zhang (1994) by using the Malmquist index in empirical growth studies.

[3]Because the equivalence between the two productivity measures have been studied by Caves, et al. (1982), this study focuses on *observational equivalence*, which roughly implies *similarity*.

index in the standard RBC model.

Technology shocks play an important role in business cycle research because most RBC models rely primarily on technology shocks to explain the fluctuations of key macroeconomic variables in the absence of demand shocks.[4] In fact, the most RBC models have been quite successful to generate artificial data series, whose characteristics are similar to the ones from the observed U.S. data.

Since the earliest equilibrium business cycle research conducted by Kydland and Prescott (1982) and Long and Plosser (1983), Solow's growth accounting approach has been mainly used to characterize the nature of the technology shocks in the business cycle studies. As Prescott (1986) pointed out, the stochastic components of the technology shocks measured by the Solow residual were critical for the success of RBC models. Prescott (1986) made a random walk assumption on the aggregate technology shocks, while King and Rebelo (1999) assumed a trend-stationary process. Ireland (2001) considered both specifications of technology shocks and argued in favor of the model of trend stationary shocks. Others, including Kollmann (1996), Baxter and Crucini (1995), Backus, Kehoe, and Kydland (1992), and Kehoe and Perri (2002) estimated the technology shock parameters in the international business cycle context. All these estimates revealed a high degree of persistence and some volatility of the technology shocks. While previous researchers have often made slightly different assumptions about the stochastic characteristics about the technology shocks, they have not made any considerable deviations from the traditional Solow's growth accounting framework when estimating the technology shocks.

In this paper, the Malmquist index is presented as an alternative technology shock mea-

[4] In fact, Ireland (2004) argued the demand shocks such as monetary policy shocks and markup shocks were important in a basic New Keynesian model for explaining the fluctuations. However, his model like the basic New Keynesian model does not have capital, which ignores the the potential propagation channel that the technology shocks are important. Erceg, Guerrier, and Gust (2005) showed that technology shocks play an important role in sticky-price (New Keynesian) model with variable capacity utilization, costs of adjustment for investment, and habit formation in consumption.

sure. Indeed, it could be potentially useful in business cycle studies when further decompositions of the technology shocks are needed to identify the sources of the shocks. According to the framework studied by Färe, Grosskopf, Norris, and Zhang (1994), the aggregate technology shock measured by the Malmquist index can be easily decomposed into two parts: technological change and efficiency change.[5] Thus, the Malmquist index could be instrumental in studying the implications of the sources of the technology shocks in business cycle studies. The last section of the paper will elaborate this point.

Despite its advantages, the Malmquist index has been overlooked by existing business cycle researchers. This study begins with the presentation of a prototype RBC model and a brief illustration of the key aspects of the model. Then, the technology shock parameters are selected solely on the basis of the Malmquist index without relying on the Solow residual. Prior to the parameterization of the technology shocks, this paper examines the empirical validity of the equivalence proposition for the two technology shock measures (the Solow residual and the Malmquist index) studied by Caves, et al. (1982). More specifically, this study shows the *observational equivalence* of the Malmquist index and the Solow residual on the basis of a non-parametric permutation tests. Finally, this study demonstrates that the model of the Malmquist index successfully replicates the standard features of the observed U.S. business cycles documented in the existing literature.

The intuition behind the main results is that the Malmquist index and the Solow residual are *observationally equivalent* to an empirical researcher with data on aggregate output and aggregate input because those two indices are constructed on the basis of the subset of the equivalence conditions suggested by Caves, et al. (1982). While the *equivalence* should hold when all the conditions are met, the results from the test indicate that even under the less than full conditions often used in the literature, the measured Malmquist index and

[5]The Malmquist index can be decomposed into more than two parts. See Kumar and Russell (2003) for details.

the Solow residual are indistinguishable to the empirical researcher. In other words, the Malmquist index can be empirically substitutable to the Solow residual. Thus, one can use the Malmquist index whenever the Solow residual is used, including in the business cycle analysis.

The rest of this paper is organized as follows. In the next section a model economy is presented. Section 3 provides a brief background of the equivalence proposition of the two aggregate technology shock measures and then presents a testing procedure for the observational equivalence. Section 4 quantitatively evaluates the model. Section 5 discusses possible extensions and application of this study and concludes.

2 Model

This study begins with outlining a standard RBC model, whose structure is similar to the simplified version of the one use by King, Plosser, and Rebelo (1988) .

2.1 The environment

The economy has a large number of identical households whose preferences are specified as follows:

$$U(C_t, N_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} N_t^{-\eta},$$

where C_t is the consumption, N_t is the labor, γ governs the intertemporal elasticity of consumption, and η is the parameter of the labor supply ($\eta > 0$).

According to King, et al. (1988), when consumption and leisure are additively separable, $\gamma = 1$ is required to get a balanced growth path. To obtain a time separable utility function between consumption and labor, this study assumes that the intertemporal elasticity of consumption is equal to 1. Given the parameter value ($\gamma = 1$), the household problem becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ln(C_t) - \eta ln(N_t)\}, \tag{1}$$

where β is the discount factor ($0 < \beta < 1$) and E_0 denotes the expectation operator, conditional on information available at time zero. The representative household is endowed with one unit of time.

The output of the economy depends on the labor and capital inputs according to a constant returns to scale production function. In particular, this study assumes a Cobb-Douglas production function,

$$Y_t = A_t K_t^{\alpha} (N_t)^{1-\alpha}, \tag{2}$$

where Y_t is the output, K_t is the capital stock, N_t is the labor, and α is the capital share. A_t is the technology shock. There are no adjustment costs associated with investment and the capital stock accumulation equation is given by

$$K_{t+1} = (1 - \delta) K_t + I_t, \tag{3}$$

where I_t is the investment and δ is the depreciation rate ($0 < \delta < 1$). There is no government in this economy and the output is used for either consumption or investment. Thus, the resource constraint is given by

$$Y_t = C_t + I_t. \tag{4}$$

This study assumes that the aggregate productivity A_t is composed of a trend growth component and a stochastic component. In particular, the law of motion governing A_t is $A_t = (1+g)^t z_t$, where g is the average rate of labor-augmenting technological progress. More specifically, $ln(z_t)$, the deviation of the natural logarithm of A_t from a linear deterministic trend, follows an AR(1) process.

$$ln(A_t) = t\ ln(1+g) + ln(z_t), \tag{5}$$

and

$$ln(z_t) = \rho ln(z_{t-1}) + \epsilon_t, \tag{6}$$

where ϵ_t, the serially uncorrelated innovation has a mean zero and a standard deviation $\sigma > 0$. In this specification, the persistence of the shock is denoted by $-1 < \rho < 1$ and the size of the shock is measured by the volatility parameter σ.[6]

2.2 Competitive Equilibrium

A competitive equilibrium is defined as a sequence of choice variables $(Y_t, C_t, I_t, K_t, N_t, R_t)_{t=1}^{\infty}$, wherein firms maximize their profits given their production technology, and households maximize their expected utilities subject to their budget constraints. In the equilibrium, all markets are cleared and the resource constraint is binding.[7]

In general, a closed-form solution of the model is not easy to obtain, but an approximate solution can be found based on a set of stationary variables. Thus, a stationary equilibrium is often characterized by log-linearizing the key equations at the steady-state values.[8] The steady state is then completely described by the structural parameters of the model and the technology shock parameters. In the absence of the shocks, the economy converges to the steady-state growth path, in which key variables become constant over time.

[6]This implies that technology shock is trend stationary, which is consistent with what Ireland (2001) found.

[7]The purpose of this section is to briefly present the underlying structure of the business cycle model. Thus, the details of the necessary conditions and their associated derivations are not discussed here. For an excellent survey of business cycle models in general, see King and Rebelo (1999).

[8]Before the log-linearizations, the logarithmic deviations of the variables from their steady state levels are considered to transform the variables. For a solution technique, see Uhlig (1999).

Once the equilibrium is computed, the model is quantitatively assessed by introducing the technology shock. Usually, the standard RBC model is evaluated using the transition dynamics of the model with the shock. Because the RBC model such as the one used in this study, is, in general, lack of an internal propagation mechanism, the characteristics of the shock are essential for the quantitative effectiveness of the model.

3 Measured Technology Shocks

Prior to the investigation of the behavior of RBC the model based on the Malmquist index, this study compares two technology shock measures — the Solow residual and the Malmquist index. While this paper is primarily interested in examining the quantitative effectiveness of the Malmquist index in the RBC model, a careful comparison could provide a justification for using the Malmquist index in the business cycle analysis, where Solow residual is predominantly used.

3.1 Equivalence: Background

This section provides a brief background information about the Malmquist index. Caves, et al. (1982) studied the economic theory behind these two productivity indices and showed that the Malmquist index could be equal to the Solow residual under certain conditions.

First, to construct the Malmquist index, one needs to define an output distance function in period t as T_t,

$$T_t(y,x) \equiv \min_\omega\{\omega : G_t(\frac{y}{\omega},\hat{x}) \leq x^1\}, \tag{7}$$

where $\hat{x}$ is the input vector excluding the first input x^1, y is the output, and $x^1 = G_t(y,\hat{x})$ is the input requirement function. The distance function measures the minimum ω required to deflate the output y, given the levels of the output y and the other input $\hat{x}$.

Now, consider changes in technology in terms of the differences in maximum output, given the input levels. Then, the Malmquist productivity index between period t and s becomes

$$M_t(x_s, x_t, y_s, y_t) \equiv \frac{T_t(y_s, x_s)}{T_t(y_t, x_t)}. \tag{8}$$

Because $T_t(y_t, x_t) = 1$ based on the definition of the distance function, Equation (8) can be rewritten as

$$M_t(x_s, x_t, y_s, y_t) = T_t(y_s, x_s) \equiv \min_\omega\{\omega : G_t(\frac{y_s}{\omega}, \hat{x}_s) \leq x_s^1\}, \tag{9}$$

where $M_t(x_s, x_t, y_s, y_t)$ measures the minimal output deflation factor required to deflate the output at time s so as to be on the production surface at time t, given the input vector of the time s.

With this definition, this study considers two inputs (capital and labor) and one output case. Following in Caves, et al. (1982), let lng_t be a translog distance function, and then assume that it is linearly homogenous in the input vector x and the output y. Then, the Malmquist index is equivalent to the Solow residual and both measure changes in true technology P_t^*. To see this, [9]

[9] For a detailed proof, see Caves, et al. (1982).

$$\textbf{Malmquist Index,} \quad P_t^*$$

$$= \overbrace{\frac{1}{2}lnM_t(x_s, x_t, y_s, y_t) + \frac{1}{2}lnM_s(x_s, x_t, y_s, y_t)}^{\text{Geometric Mean of Malmquist Productivity Indices}} \tag{10}$$

$$= \frac{1}{2}[lnT_t(y_s, x_s) - lnT_t(y_t, x_t)] + \frac{1}{2}[lnT_s(y_s, x_s) - lnT_s(y_t, x_t)] \tag{11}$$

$$= \frac{1}{2}[lng_t(y_s, x_s) - lng_t(y_t, x_t)] + \frac{1}{2}[lng_s(y_s, x_s) - lng_s(y_t, x_t)] \tag{12}$$

$$\begin{aligned} &= \{\nabla_{lny} lng_t(y_t, x_t) + \nabla_{lny} lng_s(y_s, x_s)\} \cdot [lny_s - lny_t] \\ &+\{\nabla_{lnx} lng_t(y_t, x_t) + \nabla_{lnx} lng_s(y_s, x_s)\} \cdot [lnx_s - lnx_t] \end{aligned} \tag{13}$$

$$= \underbrace{(lnY_s + \alpha_* lnL_s + (1-\alpha_*)lnK_s) - (lnY_t + \alpha_* lnL_t + (1-\alpha_*)lnK_t)}_{\text{Changes in Törnqvist Indices}} \tag{14}$$

$$= \textbf{Solow Residual,} \quad P_t^*$$

where $\nabla_{lny} lng_t(y_t, x_t)$ and $\nabla_{lnx} lng_t(y_t, x_t)$ are column vectors of the partial derivatives of lng_t with respect to lny and lnx. α_* is the average of labor share and $x \equiv (L, K)$ for a two input case (capital and labor).

As seen in Equations (10)—(14), the equivalence between the two measures require not only the translog distance function but other conditions as well. In particular, Equation (11) is obtained on the basis of the definition of the distance function and Equation (12) assumes that the distance function T is a translog. Equation (13) is obtained by a translog identity, and a quadratic identity.[10] Equation (14) describes a two-input case with assumptions of

[10] In general, the second order input coefficients in the distance function are assumed constant over time.

cost-minimization, revenue-maximization, and constant returns to scale.

3.2 Data and Measured Technology Shocks

3.2.1 Data

The sample of the study comprises data during the period from 1960 to 1995 for the U.S. The data set is largely taken from Jorgensen and Yip (2001), who assembled a detailed data set of conventional inputs and output of seven industrialized countries. This study considers a single-output, two-input production technology. The output is measured by the real GDP and the two inputs are labor hours and physical capital.

3.2.2 Constructing the Malmquist Index

The method used to construct the Malmquist index is mainly built on the framework studied by Färe, et al. (1994). This study constructs the world technology frontier by using a non-parametric frontier analysis in which each observation in the sample is compared to the constructed technology frontier. The Malmquist index is then calculated. Rather than specifying a particular aggregate production function, this study estimates the underlying technology frontier by using the Data Envelopment Analysis (DEA).[11] When solving a linear programming problem, this study makes no assumptions regarding the functional form of the reference technology. Thus, the present approach deviates from the framework suggested by Caves, et al. (1982) without relying on the translog distance function. Once the technology frontier is constructed, the Malmquist index ($P_{m,t}$) is obtained as in Equation (10).

See Diewert (1976) and Lau (1979) for details.

[11] To construct the technology frontier on basis of the DEA, data from at least two countries are required. The efficiency calculations in this study are carried out using the Data Envelopment Analysis Program software developed by Tim Coelli.

3.2.3 Constructing the Solow residual

The Solow residual is constructed on the basis of the standard growth accounting framework followed by Solow (1957) who considered a formula for total factor productivity (TFP) as a discrete approximation to the continuous divisia index. This approach also deviates from the framework suggested by Caves, et al. (1982) because it does not rely on the translog distance function. From the production function described in Equation (2), the Solow residual ($P_{s,t}$) can be constructed as in Equation (14).

3.3 Observational Equivalence

The equivalence conditions given by Caves, et al. (1982), considerably restrict empirical applications. Indeed, it may be difficult for an empirical researcher to construct both the technology shock measures using all the conditions required by the theory and to empirically show the equivalence of the two measures. In order to investigate the empirical validity of the equivalence, this study assumes that the measured technology shock contains a random noise term ε. More specifically, in each period t,

$$P_{i,t} = P_t^* + \varepsilon_{i,t} \qquad i = m, s, \tag{15}$$

where $P_{i,t}$ represents the measured technology shock for i, and P_t^* (*without a subscript* i) represents the true technology shock, which is common for both producibility measures. $\varepsilon_{i,t}$ is a random noise term. m and s represent the Malmquist index and the Solow residual, respectively.

In this specification, both the Malmquist index and the Solow residual include the true technology shock (P_t^*) and a random disturbance term (ε) as follows: $P_{m,t} = P_t^* + \varepsilon_{m,t}$ for the Malmquist index; $P_{s,t} = P_t^* + \varepsilon_{s,t}$ for the Solow residual. The random disturbance term represents possible specification errors and other unobserved factors that could cause discrepancies between the measured technology shock and the true technology shock. As described in the previous section, this study deviates from the framework suggested by Caves, et al. (1982) when it constructs the Malmquist index and the Solow residual. And such deviation justifies the non-zero disturbance term, $\varepsilon \neq 0$. The equivalence of the two technology shock measure should hold when $\varepsilon_{m,t} = \varepsilon_{s,t}$, which this study attempts to test.

To conduct a formal hypothesis test, this study further assumes that ε_i has a probability distribution $f(\varepsilon_i; \theta_i)$ with a parameter θ_i, where $i = m,\ s$. In particular, U is defined as the probability distribution for m and W is the probability distribution for s. Finally, this study adopts Rothenberg's (1971) definition of the *observational equivalence* as follows:

Definition: Two parameters θ_m and θ_s are said to be *observationally equivalent* if $U(P; \theta_m) = W(P; \theta_s) \quad for\ all\ P \in\ \mathbb{R}^n$

3.3.1 Permutation Tests

In order to examine the observational equivalence, this study employs a non-parametric permutation test, which generally requires fewer assumptions than traditional methods do. No distributional assumptions are required for the test and the validity of the test depends solely on the randomization. An appealing feature of the permutation test is that the mean, the median, or any other test statistics can be used to obtain exact calculations of significance levels.

This study follows Efron and Tibshirani's (1993) algorithm to conduct the permutation test. More specifically, let $P_{i,m}$ and $P_{i,s}$ be the ith observation of the Malmquist index and the Solow residual respectively. An empirical researcher observes $P_m = (P_{1,m}, \cdots, P_{j,m})$ and $P_s = (P_{1,s}, \cdots, P_{k,s})$, drawn from possibly different probability distributions U and W: j and k represent the number of observations. The researcher wishes to test the null hypothesis

H_o of no difference between U and W,

$$H_o: \quad U = W.$$
$$H_a: \quad \mathrm{H_o} \text{ is not true.}$$

If H_o is true, there is no difference between the probabilistic behavior of P_m and P_s. Thus, the two technology shock measures can be randomized. Under the null hypothesis, the conditional distribution of the observations — given their combined ordered statistics — is permutation invariant.[12] In this case, a non-rejection of the null hypothesis implies that the Malmquist index and the Solow residual are observationally equivalent to the empirical researcher. Taking advantage of the permutation test, this study selects four different test statistics $\phi = \theta_m - \theta_s$: the mean difference, the median difference, the first quartile difference, and the third quartile difference.

Using the resampling technique, this study considers 1,000 resamples, each of which is divided into two groups.[13] It then constructs the permutation distribution of the each test statistic, $\hat{\phi}$. Under the null hypothesis, no statistic from the two groups should exhibit any differences, that is $\phi = 0$. Having observed $\hat{\phi}^*$ from the original data, the Achieved Significance Level (ASL) of each permutation test (for $\hat{\phi}^* > 0$ case) is computed as follows:

$$A\hat{S}L = Prob(\hat{\phi} \geq \hat{\phi}^*) = \frac{\#(\hat{\phi} \geq \hat{\phi}^*)}{(j+k)!/(j!k!)}, \tag{16}$$

where # indicates the number of times.[14] The smaller the value of the ASL, the stronger is the evidence against the null hypothesis.

[12] For details about randomization tests, see Kennedy (1995) or Good (2000).

[13] This study considers 1,000 replications, which is the minimum number recommended by Efron and Tibshirani (1993). In fact, the number of replications do not affect the main results of the study. The permutation tests on the basis of 100,000 replications provide qualitatively very similar results.

[14] In most cases, the number of possible randomizations is considerably large ($\frac{70!}{35! \times 35!} = 1.1219 \times 10^{20}$ combinations). Thus, this study uses the Monte Carlo methods to approximate the ASL.

Table 1 presents the test results. The findings suggest that the measured technology shocks by Malmquist index and the Solow residual are observationally equivalent. In particular, at 10% and 5% levels of significance, this study does not find statistically significant evidence against the null hypothesis on the basis of the means, the medians, the first quartiles, and the third quartiles.[15] In addition, the conventional t-test also shows the same result.

The results of those tests suggest that the observed differences between the two technology shock measures, $\hat{\phi}$, may be negligible. In other words, the Malmquist index and the Solow residual are *observationally equivalent* to an empirical researcher with data on the aggregate output and aggregate input. As a consequence, the empirical validity of the equivalence proposition for the two technology shock measures is assured.

4 Model Evaluations

While it is the Solow residual that is overwhelmingly used in most RBC models as a proxy for measured technology shock, the observational equivalence shown in the previous section suggests that the Malmquist index is empirically compatible to the Solow residual. With this in mind, this paper evaluates the effectiveness of the RBC model described in Section 2. In particular, this study compares the unconditional second moments of the observed data with those of the series generated from the model. For this comparison, it examines the impulse responses to the technology shock based on the Malmquist index along with those based on the Solow residual.

[15] This is, in principle, consistent with what Van Biesebroeck (2006) documented using the firm level data from Columbia and Zimbabwe. He showed different methods of measuring productivity (including those two used in this study) produced similar productivity estimates.

4.1 Calibration

The calibration of the structural parameters of the model follows the standard RBC literature. As in King and Rebelo (1999), the real return to capital is set to 6.5%, which equals to the average annual real return to the S&P 500 so that the discount factor β becomes 0.9389. The intertemporal elasticity substitution of consumption is set to 1. The annual depreciation rate is assumed to be 10%, and the labor share is set to 64% as in Prescott (1986). Finally, the representative household devotes 30% of its time endowment to the market.

For complete model evaluations, two aggregate technology shock parameters needs to be selected: the persistence parameter ρ and the volatility parameter σ. To estimate them, this study constructs two sets of the natural logarithm of A_t: one based on the Solow residual and the other based on the Malmquist index. Then, as in King and Rebelo (1999), a linear trend is fitted to the $ln(A_t)$ to compute the trend growth rate g as shown in Equation (5). Finally, using the residuals from this regression, estimates ρ and σ of ϵ are obtained.

The estimated persistence parameter of the technology shock based on the Malmquist index ρ_M is 0.958 and the estimated volatility parameter σ_M is 1.629.[16] The estimated persistence parameter based on the Solow residual ρ_S is 0.958 and the estimated volatility parameter σ_S is 1.629. The calibration of the model is summarized in Table 2.

As seen in Table 2, the persistence of the measured technology shock based on the Malmquist index is slightly bigger than that based on the Solow residual. And the standard deviation the measured technology shock based on the Malmquist index is slightly smaller than that based on the Solow residual. Because the technology shock based on the Malmquist index is more persistence, it is reasonable to have a smaller volatility of it. Overall, there are no sizable differences between the two sets of aggregate productivity shock parameters.

[16]Because the technology shock parameters based on the Malmquist index have never been considered in the RBC literature, there are no documented values to compare to. Nevertheless, they are not much different from the other set of parameter values based on the Solow residual.

Figure 1 plots the comparison of the two alternative measures.

4.2 Cyclical Behavior

In this section, the calibrated model is simulated by introducing technology shocks and its effectiveness is evaluated on the basis of the second moments of the cyclical component of the generated series. The cyclical part of each variable is obtained after the application of the Hodrick-Prescott filter, which eliminates the smooth trend from the data. More specifically, this study compares the second moments of the series generated from the model with those of the observed data. Table 3 presents the results. Panels I, II, and III in Table 3 show the standard deviations, their associated standard errors (only for the artificial series), the relative standard deviations (to output), and the contemporaneous correlations (with output) for the U.S. data and the artificial data obtained from the two models.

Panel II in Table 3 shows the second moments of the key variables generated from the model based on the Malmquist index. First, the model economy successfully replicates the key features of the observed data. The business cycle statistics are consistent with those already documented in the existing literature. In the model economy, the most of the variables fluctuates slightly less than those observed in the U.S. economy. The investment generated in the model fluctuates much more than the output and the consumption in the model economy is smoother than he output. In addition, consumption, investment, and labor in the model economy are all procyclical; the contemporaneous correlation coefficients between each key variable and the output are all greater than 0.97. Finally, the contribution of the aggregate productivity to the output measured by the relative standard deviation is approximately 70% in the model economy.[17] As the standard model predicts, the aggregate productivity is a major contributor to business cycles.

[17] On the basis of quarterly data, McGrattan (2005) documented that the average contribution of the aggregate productivity to the output spectrum was about 70%, which is generally considered the upper bound in the absence of other non-technology shocks in this line of research.

As a benchmark case, the model based on the Solow residual is simulated; and except for the frequency of data, the result is a textbook case of the RBC model. Panel III in Table 3 presents the relevant statistics, which can be compared to those in Panel II. The characteristics of the series generated from the model based on the Solow Residual are very similar to those of the series based on the Malmquist index. The rankings of the standard deviations and the extent of the relative standard deviations in the two models are very similar. The standard errors of the second moments, which are computed using 1,000 simulations, indicate that the marginal differences between two sets of the point estimates of the standard deviation could be the result of sampling errors. All the variables are procyclical and the aggregate productivity accounts for approximately 68% of the movement in the output.

4.3 Impulse Responses

The impulse responses further verify the quantitative effectiveness of the model based the Malmquist index. Figure 2 shows the expected responses of key variables to a technology shock, beginning from the steady-state level, where the aggregate technology increases by 1% in the initial period. The two sets of impulse responses reveal a remarkable resemblance between these two models.

The results suggest that the RBC model based on the Malmquist index is successful in replicating the results of the standard business cycle based on the Solow residual. An increase in the aggregate technology increases the consumption demand on impact. However, the consumption increases to a lesser extent than the output because the households want to smooth their consumption. Thus, the households would save more in the early periods. Consequently, investment increases. Finally, the employment increases with the aggregate productivity shock because of substitution effects dominate. When the aggregate technology returns to its original level, the economy slowly scales down its excess capital stock. Figures 3–5 present additional impulse functions for these variables by providing direct comparisons between two aggregate technology measures.

5 Discussions and Concluding Remarks

This paper presents an alternative method of choosing the aggregate technology shock parameters – without using the Solow residual – in a standard RBC model. By doing so, this study seeks to evaluate a quantitative role of the Malmquist index in business cycle studies. Although the Malmquist index has certain advantages, few researchers have studied its implications on the business cycle.

On the basis of permutation tests, this study first investigates the empirical validity of the equivalence proposition on the two technology shock measures: it finds the observational equivalence of the Malmquist index and the Solow residual. It then studies the role of technology shocks in the RBC model based on the Malmquist index. The findings suggest that the RBC model based on the Malmquist index is successful not only in generating U.S.-like business cycles but also in producing results comparable to those of the RBC model based on the Solow residual. Thus, the Malmquist index is as important as the Solow residual in terms of its quantitative relevance to business cycle research.

The recent literature on the equilibrium business cycle goes beyond the initial framework presented by Kydland and Presott (1982). Indeed, as seen in Parente and Prescott (2000) and Lagos (2004), current research has started to examine the sources of variation in the aggregate technology. While the main focus of the paper is to compare the Malmquist index with the Solow residual, one could also examine the underpinnings of aggregate technology using the Malmquist index under a more richer model setup than the one presented here, because the Malmquist index, in principle, allows one to decompose technology shocks relatively easily.

For instance, the Malmquist can be decomposed into two components: efficiency change (EC) and technological change (TC). In other words, the Malmquist index measures technology shocks relative to piece-wise linear frontier production functions simultaneously accounting for efficiency change below the frontier function and shifts of the frontier function itself. To see this,

$$\begin{aligned}\text{Malmquist}_{t,t+1}(y_{t+1}, x_{t+1}, y_t, x_t) &= EC \cdot TC \\ \text{where } \; EC &= \frac{T_{t+1}(x_{t+1}, y_{t+1})}{T_t(x_t, y_t)}, \\ \text{and } \; TC &= \Big[\frac{T_t(x_{t+1}, y_{t+1})}{T_{t+1}(x_{t+1}, y_{t+1})} \times \frac{T_t(x_t, y_t)}{T_{t+1}(x_t, y_t)}\Big]^{\frac{1}{2}}\end{aligned}$$

In this decomposition, EC would be improvements in efficiency, which measures the distance of the production plan relative to the best production technology frontier. Thus, if a researcher would like to introduce nominal or real frictions into the standard equilibrium business cycle models, which would cause inefficiency within the economy, the Malmquist index could be a useful tool to quantify the effects of the technology shocks and to study implications of those frictions in the business cycle analysis. Such examples include short run inefficiency caused by monopolistic competition in standard sticky price models as in Woodford (2003); investment and capital adjustments costs causing sluggish adjustments as in Edge (2007); costly reallocation producing uneven impacts of technology shocks as in Shapiro and Ramey (1998), etc. For all these cases, the production plan of the economy could be inside the best production technology frontier.[18]

Another area where the decomposition of the Malmquist index could potentially useful would be the measurement issues associated with Solow residual as aggregate technology. Deviating from the standard growth accounting approach, Basu, Fernald, and Kimball (2006)

[18] In fact, the equilibrium if exists, it could still be constrained efficient, where the constraints are imposed by the model

measured technology shocks based on a cost minimization, which controlled for imperfect competition, varying utilization of capital and labor, and aggregation effects. And then they called their technology measures as "purified technological progress" because they are highly refined estimate of technology change, where all the "non-technology" components of Solow residuals are netted out. Indeed, the documented that the cyclical behavior of the purified technological progress were quite different from that of the traditional Solow residual. They along with Galí (1999) argued against the RBC model by looking at the relationship between measured technology shocks and labor inputs.[19] Interestingly, their findings are consistent with sticky-price models, which non-technology shocks such as monetary shocks are important on business cycle. In this line of research, the role of measured technology shocks on the business cycle has been a deciding criteria for the model selection, between the RBC models and the sticky-price models. Because the measured technology shocks play critical roles in this debate, alternative technology shock measure, the Malmquist index and its decompositions could potentially help to reconcile different views among business cycle researchers.[20]

While it has been about 25 years since the new methods proposed by Kydland and Prescott (1982), there are still open questions on the role of technology shocks on the business cycles. And the findings of this study could provide another angle to look at the technology shocks in the RBC model and could open the door to exciting further business cycle research based on the Malmquist index. The paper leaves the unsolved issues for future research.

[19]Basu et al. (2006) found that in the short run, technology improvements reduce input use, which is inconsistent with the prediction made by the standard RBC model. Galí (1999), on the other hand, used a long-run restriction on a structural vector autoregression (SVAR) to find the same conclusion.

[20]See also Francis and Ramey (2005), Christiano, Eichembaum, and Vigfusson (2003) and Chari, Kehoe, and McGrattan (2007) for on-going debates on the role of technology based on SVAR with the long-run restrictions.

References

[1] Basu, S., J. Ferland, and M. Kimball (2006), Are Technology Improvements Contractionary?, *American Economic Review*, 96, 1418–1448.

[2] Baxter, M. and M. Crucini (1995), Business Cycles and the Asset Structure of Foreign Trade, *International Economic Review*, 36, 821–854.

[3] Backus, D., P. Kehoe, and F. Kydland (1992), International Business Cycles, *Journal of Political Economy*, 101, 745–775.

[4] Caves, D., L. Christensen, and W. Diewert (1982), The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity, *Econometrica* 50:6, 1393–1414.

[5] Chari, V., P. Kehoe, and E. McGrattan (2008), Are Structural VARs with Long-Run Restrictions Useful in Developing Business Cycle Theory? *Journal of Monetary Economics*, 55:8, 1337–1352,

[6] Christiano, L., M. Eichenbaum, and R. Vigfusson (2003), What Happens After a Technology Shock? *NBER Working Paper*, #9819.

[7] Coelli, T. (1996), A Guide to DEAP: A Data Envelopment Analysis (Computer) Program, University of New England.

[8] Diewert, W. (1976), Exact and Superlative Index Numbers, *Journal of Econometrics*, 4, 115–145.

[9] Edge, R. (2007), Time to Build, Time to Plan, Habit Persistence, and the Liquidity Effect, *Journal of Monetary Economics*, 54, 1644–1669.

[10] Efron, B. and R. Tibshirani (1993), *An Introduction to the Bootstrap*,Norwell, MA: Chapman & Hall/CRC.

[11] Erceg, C., L. Guerrieri, and C. Gust (2005) Can Long-Run Restrictions Identify Technology Shocks? *Journal of European Economic Association*, 3, 1237–1278.

[12] Francis, N. and V. Ramey (2005). Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited, *Journal of Monetary Economics*, 52, 1379–1399.

[13] Färe, R., S. Grosskopf, M. Norris, and Z. Zhang (1994), Productivity Growth, Technical Progress, and Efficiency Change in Industrial Countries, *American Economic Review*, 84, 66–83.

[14] Gáli, J. (1999), Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *American Economic Review*, 89, 249–271.

[15] Good, P. (2000), *Permutation tests*: A Practical Guide to Resampling Methods for Testing Hypothesis, New York: Springer-Verlag.

[16] Ireland, P. (2001), Technology Shocks and the Business Cycle: An Empirical Investigation, *Journal of Economic Dynamics and Control*, 25, 703–719.

[17] Ireland, P. (2004), Technology Shocks in the New Keynesian Model, *Review of Economics and Statistics*, 86, 923–936.

[18] Jorgenson, D. and E. Yip (2001), Whatever Happened to Productivity Growth? in: E. Dean, M. Harper, and C. Hulten, eds., *New Developments in Productivity Analysis*, 205–246, Chicago, IL: University of Chicago Press.

[19] Kehoe, P. and F. Perri (2002), International Business Cycles with Endogenous Incomplete Markets, *Econometrica*, 96, 921–947.

[20] Kennedy, P. (1995), Randomization Tests in Econometrics, *Journal of Business and Economic Statistics*, 13, 85–94.

[21] King, R. and S. Rebelo (1999), Resuscitating Real Business Cycles, 927–1007, in: J. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Amsterdam: Elsevier Science Publishers.

[22] King, R., C. Plosser, and S. Rebelo (1988), Production, Growth and Business Cycles: I. The Basic Neoclassical Model, *Journal of Monetary Economics*, 21, 195–232.

[23] Kollmann, R. (1996), Incomplete Asset Markets and the Cross-Country Consumption Correlation Puzzle, *Journal of Economic Dynamics and Control*, 20, 945–961.

[24] Kumar, S. and R. Russell (2003), Technological Change, Technological Catch-Up, and Capital Deepening: Relative Contribution of Growth and Convergence, *American Economic Review*, 92, 527–548.

[25] Kyland, F. and E. Prescott (1982), Time to Build and Aggregate Fluctuations, *Econometrica*, 50, 1345–1370.

[26] Lagos, R. (2006), A Model of TFP,*Review of Economic Studies*, 73, 983–1007.

[27] Lau, L. (1979), On Exact Index Numbers, *Review of Economics and Statistics*, 61, 73–82.

[28] Long, J. and C. Plosser (1983), Real Business Cycles, *Journal of Political Economy*, 91, 39–69.

[29] McGrattan, E. R. (2005), Comment on Gali and Rabanal's "Technology Shocks and Aggregate Fluctuations: How Well Does the RBC Model Fit Postwar U.S. Data?" in: M. Gertler and K. Rogoff eds., *NBER Macroeconomic Annual 2004*, 289–308, Cambridge, MA: MIT Press.

[30] Parente, S. and E. Prescott (2000), *Barriers to Riches*, Cambridge, MA: MIT Press.

[31] Prescott, E. (1986), Theory Ahead of Business-Cycle Measurement, *Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 9–22.

[32] Ramey, V. and M. Shapiro (1998), Costly Capital Reallocation and the Effects of Government Spending, *Carnegie Rochester Conference Series on Public Policy*, 48, 145-94.

[33] Rothenberg, T. (1971), Identification in Parametric Models, *Econometrica*, 39, 577–591.

[34] Solow, R. (1957), Technical Change and the Aggregate Production Function, *Review of Economic Studies*, 39, 312–320.

[35] Uhlig, H. (1999), A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily, 30–62, in: R. Marinom and A. Scott eds., *Computational Methods for the Study of Dynamic Economies*, Oxford: Oxford University Press.

[36] Van Biesebroeck, J. (2006), The Sensitivity of Productivity Estimates: Revisiting Three Important Debates, *Journal of Business and Economic Statistics*, forthcoming.

[37] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

Table 1: Observational Equivalence Test Results

	Permutation Tests (ASLs)				t-test (P-value)
Test Statistics	Mean	Median	1st Q	3rd Q	Mean
	0.383	0.432	0.653	0.623	0.743

Note: The results are based on the two-tailed tests. ASL stands for the achieved significance level. 1st Q and 3rd Q represent the first quartile and the third quartile, respectively. The permutation tests are based on 1,000 replications.

Table 2: Calibration Summary

Parameters	Value	Interpretation
β	0.939	Subjective Discount Rate
δ	0.100	Depreciation Rate of Capital
$\bar{R}$	1.065	Real Return to Capital
η	1.000	Intertemporal substitution
$\bar{N}$	0.300	Steady State Employment
$\bar{A}$	1.000	Scale Parameter of Aggregate Productivity
α	0.360	Share of Capital
Malmquist Index		
ρ_M	0.958	Persistence Parameter
σ_M	1.629	Volatility Parameter
Solow Residual		
ρ_S	0.928	Persistence Parameter
σ_S	1.759	Volatility Parameter

Table 3: Key Business Cycle Statistics

Variables	St. Dev		Rel. St. Dev	Corr with y
Panel I (U.S. Data)				
y	2.00		1.00	1.00
c	1.77		0.89	0.88
i	7.92		3.96	0.81
n	1.79		0.90	0.88
A_S	1.71		0.86	0.71
A_M	1.55		0.78	0.59
Panel II (Model with Malmquist Index)		S.E.		
y	1.44	0.098	1.00	1.00
c	0.71	0.052	0.49	0.97
i	4.17	0.282	2.89	0.99
n	0.76	0.052	0.52	0.97
A_M	1.01	0.069	0.70	1.00
Panel III (Model with Solow Residual)		S.E.		
y	1.64	0.109	1.00	1.00
c	0.76	0.053	0.49	0.96
i	5.00	0.330	3.04	0.99
n	0.94	0.019	0.57	0.97
A_S	1.11	0.074	0.68	1.00

Note: All variables are in logarithms and have been de-trended with the Hodrick-Prescott filter. y is the per capita output, c is the per capita consumption, i is the per capita investment, l is the per capita hours, A_M is the level of aggregate productivity based on the Malmquist index, and A_S is the level of aggregate productivity based on the Solow residual. *St. Dev* in the second column stands for the standard deviation. *S.E* in the third column in Panels II and III stands for the standard error of the estimated standard deviation, which is computed on the basis of 1,000 simulations. *Rel. St. Dev* in the fourth column stands for the relative standard deviation, which is computed as a ratio of the standard deviation of each variable to the standard deviation of the output. *Corr with y* in the last column presents the contemporaneous correlations of each variable with the output.

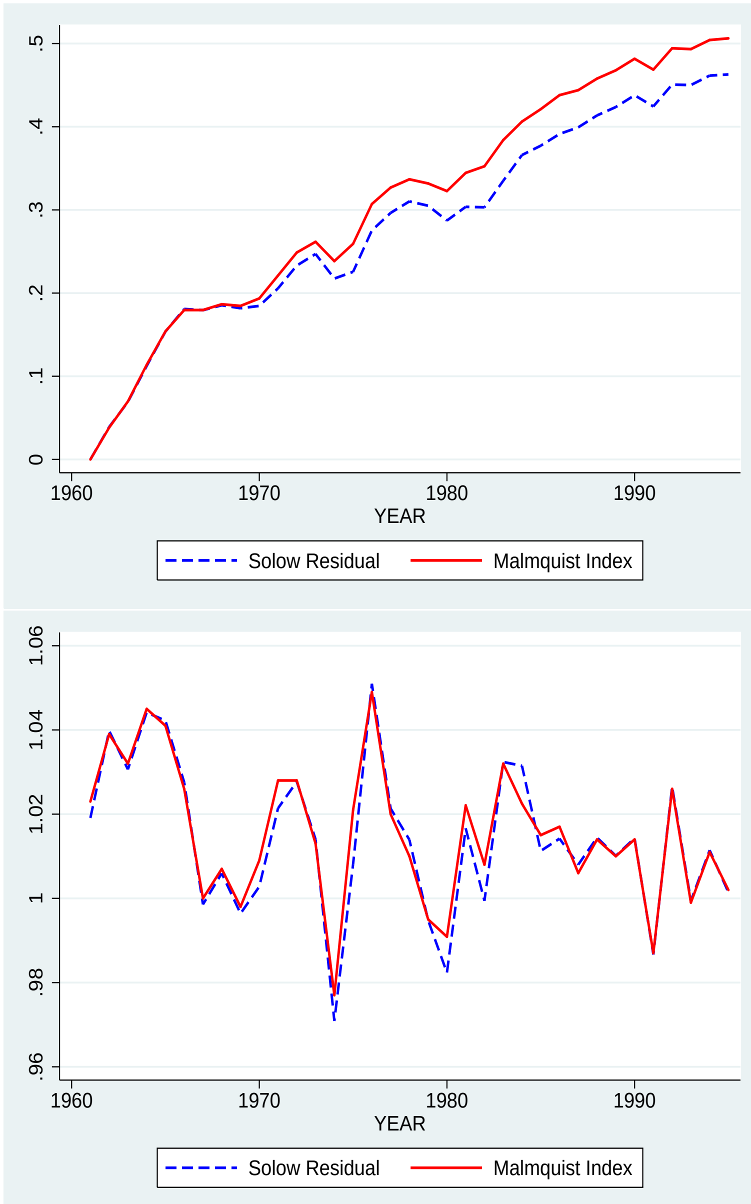

Figure 1: Solow Residual and Malmquist Index: The first plot represents the aggregate productivity levels (normalized at the beginning of the period) and the second plot represents growth rates. A value greater than one indicates a positive growth from period t to period $t+1$. A value less than one indicates a negative growth from period t to period $t+1$.

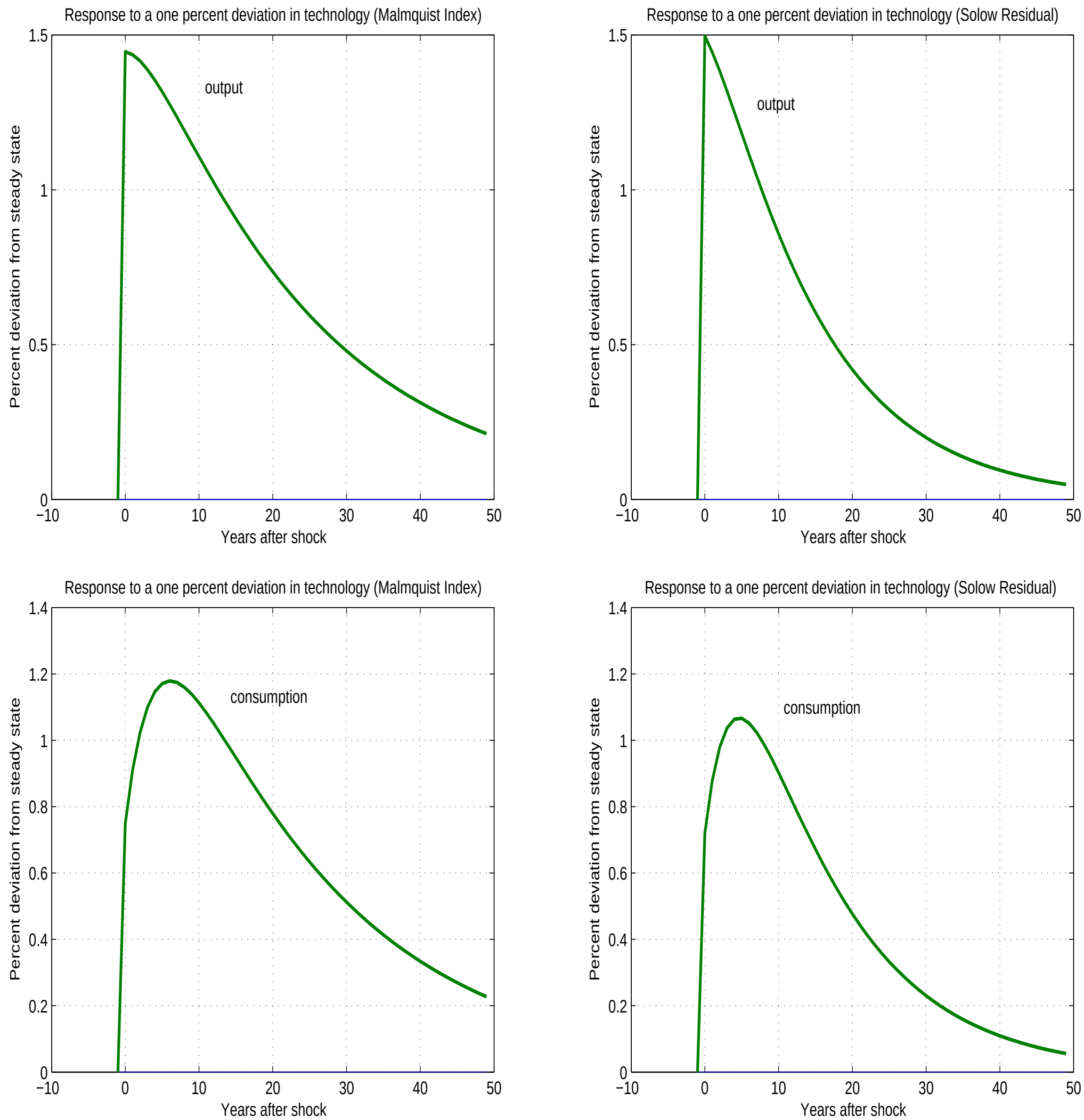

Figure 2: Impulse Responses of Output and Consumption to a Positive 1% Shock in Aggregate Technology. The upper (output) and the lower (consumption) left plots are based on the Malmquist index and the upper (output) and the lower (consumption) right plots are based on the Solow residual.

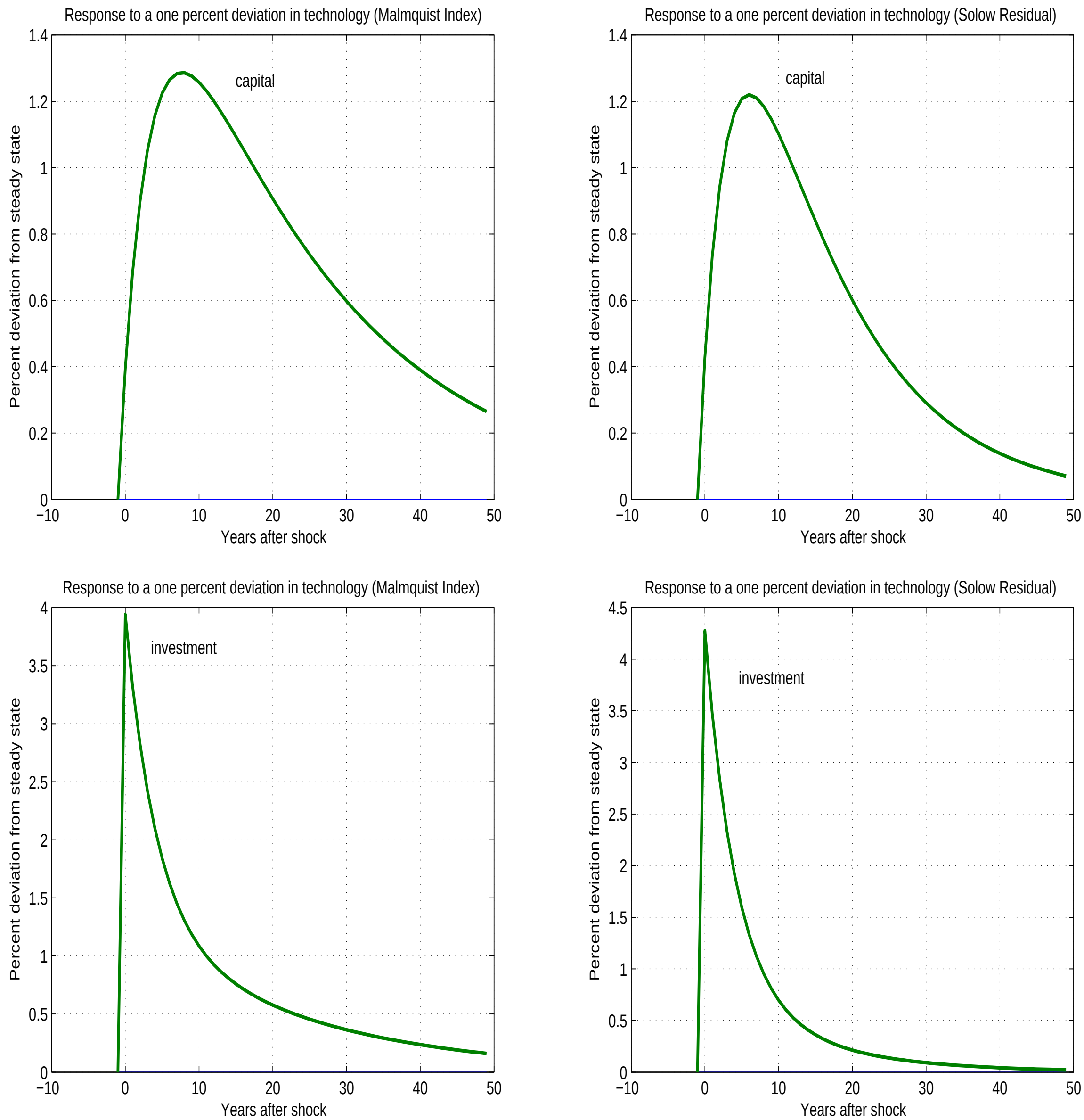

Figure 3: Impulse Responses of Capital and Investment to a Positive 1% Shock in Aggregate Technology. The upper (capital) and the lower (investment) left plots are based on the Malmquist index and the upper (capital) and the lower (investment) right plots are based on the Solow residual.

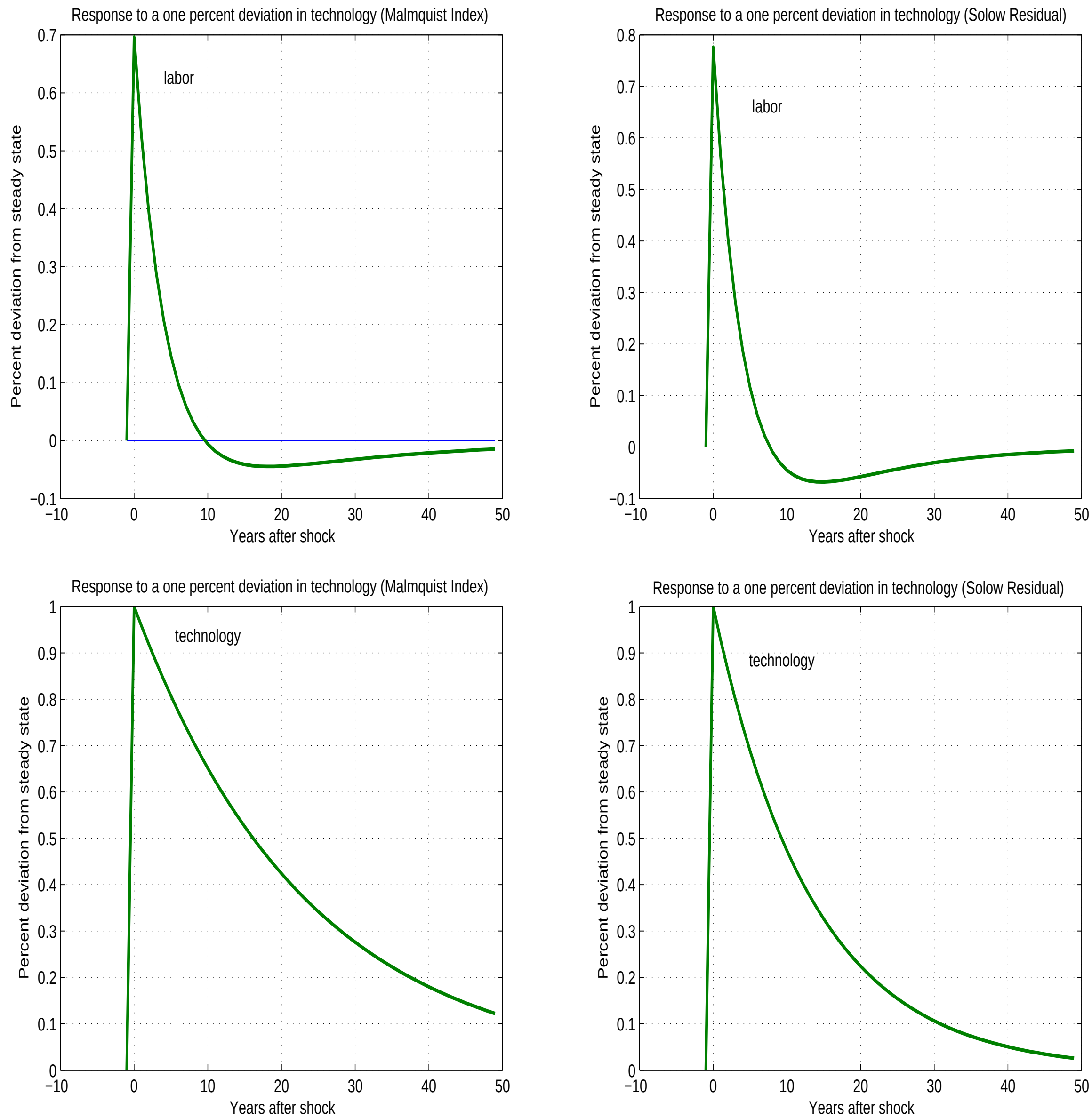

Figure 4: Impulse Responses of Labor and Aggregate Technology to a Positive 1% Shock in Aggregate Technology. The upper (labor) and the lower (technology) left plots are based on the Malmquist index and the upper (labor) and the lower (technology) right plots are based on the Solow residual.